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# Zeros of the Zak Transform of Averaged Totally Positive Functions

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## Abstract

Let  $\alpha > 0$  and let  $g \in L_1(\mathbb{R})$  be a continuous function, whose Fourier transform is

$$\widehat{g}(\omega) = Ce^{-\gamma\omega^2} e^{-2\pi i\delta\omega} \left( \prod_{\nu=1}^{\infty} \frac{e^{2\pi i\delta_\nu\omega}}{1 + 2\pi i\delta_\nu\omega} \right) \left( \prod_{j=1}^m \frac{e^{\lambda_j - 2\pi i\alpha\omega} - 1}{\lambda_j - 2\pi i\alpha\omega} \right),$$

where  $C > 0$ ,  $\gamma \geq 0$ ,  $\delta, \delta_\nu, \lambda_j \in \mathbb{R}$ ,  $\sum_{\nu=1}^{\infty} \delta_\nu^2 < \infty$ ,  $m \in \mathbb{Z}_+$ . We prove that its Zak transform  $Z_\alpha g(x, \omega) = \sum_{k \in \mathbb{Z}} g(x + \alpha k) e^{-2\pi i k \alpha \omega}$  has only one zero  $(x^*, \frac{1}{2\alpha})$  in the fundamental domain  $[0, \alpha) \times [0, \frac{1}{\alpha})$ . In particular, the result is valid for totally positive functions. Earlier it was known for such functions without the factor  $e^{-\gamma\omega^2}$ . We also establish simplicity of the zero with respect to each variable and give the applications to Gabor analysis. The described class of functions is closed under convolution.

*Keywords:* Exponential  $B$ -splines, Gabor frames, Totally positive functions, Zak transform

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## 1. Introduction

We denote by  $\mathbb{C}$ ,  $\mathbb{R}$ ,  $\mathbb{Z}$ ,  $\mathbb{Z}_+$  and  $\mathbb{N}$  the sets of complex, real, integer, nonnegative integer and natural numbers respectively;  $L_1(\mathbb{R})$  is the space of Lebesgue integrable on  $\mathbb{R}$  functions,  $\chi_E$  is the characteristic function of a set  $E$ . The Fourier transform of a function  $f \in L_1(\mathbb{R})$  is given by the formula

$$\widehat{f}(\omega) = \int_{\mathbb{R}} f(t) e^{-2\pi i t \omega} dt.$$

The convolution of two functions  $f$  and  $h$  is defined by

$$(f * h)(x) = \int_{\mathbb{R}} f(t) h(x - t) dt$$

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