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Zeros of the Zak Transform of Averaged Totally Positive Functions

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Abstract

Let $\alpha > 0$ and let $g \in L_1(\mathbb{R})$ be a continuous function, whose Fourier transform is

$$\widehat{g}(\omega) = Ce^{-\gamma\omega^2} e^{-2\pi i\delta\omega} \left(\prod_{\nu=1}^{\infty} \frac{e^{2\pi i\delta_\nu\omega}}{1 + 2\pi i\delta_\nu\omega} \right) \left(\prod_{j=1}^{m} \frac{e^{\lambda_j - 2\pi i\alpha\omega} - 1}{\lambda_j - 2\pi i\alpha\omega} \right),$$

where $C > 0$, $\gamma \geqslant 0$, $\delta, \delta_\nu, \lambda_j \in \mathbb{R}$, $\sum\limits_{\nu=1}^{\infty} \delta_\nu^2 < \infty$, $m \in \mathbb{Z}_+$. We prove that its Zak transform $Z_\alpha g(x, \omega) = \sum\limits_{k\in\mathbb{Z}} g(x + \alpha k)e^{-2\pi ik\alpha\omega}$ has only one zero $(x^*, \frac{1}{2\alpha})$ in the fundamental domain $[0, \alpha) \times \bigl[0, \frac{1}{\alpha}\bigr)$. In particular, the result is valid for totally positive functions. Earlier it was known for such functions without the factor $e^{-\gamma\omega^2}$. We also establish simplicity of the zero with respect to each variable and give the applications to Gabor analysis. The described class of functions is closed under convolution.

Keywords: Exponential B-splines, Gabor frames, Totally positive functions, Zak transform
2010 MSC: 42C15, 42C40

1. Introduction

We denote by $\mathbb{C}$, $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{Z}_+$ and $\mathbb{N}$ the sets of complex, real, integer, nonnegative integer and natural numbers respectively; $L_1(\mathbb{R})$ is the space of Lebesgue integrable on $\mathbb{R}$ functions, χ_E is the characteristic function of a set E. The Fourier transform of a function $f \in L_1(\mathbb{R})$ is given by the formula

$$\widehat{f}(\omega) = \int_{\mathbb{R}} f(t)e^{-2\pi it\omega}\, dt.$$

The convolution of two functions f and h is defined by

$$(f * h)(x) = \int_{\mathbb{R}} f(t)h(x - t)\, dt$$

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