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POWER SERIES APPROXIMATIONS TO FEKETE POLYNOMIALS

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Abstract. We study how well Fekete polynomials

$$F_p(X) = \sum_{n=0}^{p-1} \left(\frac{n}{p}\right) X^n \in \mathbb{Z}[X]$$

with the coefficients given by Legendre symbols modulo a prime p, can be approximated by power series representing algebraic functions of a given degree. We also obtain some explicit results describing polynomial recurrence relations which are satisfied by the coefficients of such algebraic functions.

1. Introduction

1.1. **Background and motivation.** For a prime $p \geqslant 3$ we recall that the *Fekete polynomial*

$$F_p(X) = \sum_{n=0}^{p-1} \left(\frac{n}{p}\right) X^n \in \mathbb{Z}[X]$$

is the polynomial of degree $p-1$ with coefficients given by Legendre symbols modulo p. These polynomials are a classical object of study, investigated initially by Fekete and Pólya [12] (see also [17]) and then by several others, Baker and Montgomery [1], Conrey, Granville, Poonen and Soundararajan [8], Erdélyi [9, 10] and Günther, and Schmidt [13]. Heilbronn [14] has also considered an infinite power series of the same shape as Fekete polynomials.

Till now, most of the attention has been devoted to analytic properties of these polynomials, such as the distribution of zeros and relations between various norms, see [8–11, 13] and references therein.

Here we consider an apparently new question about the algebraic nature of Fekete polynomials. Namely for an integer $N \geqslant 2$ we denote

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