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BOUNDS FOR THE ASYMPTOTIC ORDER PARAMETER OF THE STOCHASTIC KURAMOTO MODEL

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Dedicated to Boróka, Eszter and Koppány

ABSTRACT. Turán type inequalities for modified Bessel functions of the first kind are used to deduce some sharp lower and upper bounds for the asymptotic order parameter of the stochastic Kuramoto model. Moreover, approximation from the Lagrange inversion theorem and a rational approximation are given for the asymptotic order parameter.

1. INTRODUCTION

The Kuramoto model describes the phenomenon of collective synchronization, more precisely it describes how the phases of coupled oscillators evolve in time, see [Ku] and [BGP, p. 271] for more details. Recently Bertini, Giacomin and Pakdaman [BGP] were able to review some results on the Kuramoto model from a statistical mechanics standpoint and they gave in particular necessary and sufficient conditions for reversibility. In order to do this Bertini, Giacomin and Pakdaman [BGP, p. 278] deduced some lower and upper bounds for the asymptotic order parameter, which involves the modified Bessel functions of the first kind of order zero and one. A few years later Sonnenschein and Schimansky-Geier [SS] obtained the asymptotic order parameter in closed form, which suggested a tighter upper bound for the corresponding scaling. Moreover, they elaborated the Gaussian approximation in complex networks with distributed degrees. In their study Sonnenschein and Schimansky-Geier [SS, p. 3] proposed another upper bound for the asymptotic order parameter, but they presented their result without mathematical proof. All the same, by using Bernoulli's inequality they verified that their upper bound is better than the upper bound of Bertini, Giacomin and Pakdaman [BGP]. In this paper our aim is to make a contribution to this subject by showing the followings:

- The bounds presented in the above mentioned papers are correct and their proofs are based on some Turán type inequalities for modified Bessel functions of the first kind.
- The constants in the upper bounds presented by Bertini, Giacomin, Pakdaman [BGP] and Sonnenschein, Schimansky-Geier [SS] are the best, and thus their bounds cannot be improved.
- The results presented in the above mentioned papers can be extended to modified Bessel functions of the first kind of arbitrary order, based on some interesting new and recently discovered Turán type inequalities for modified Bessel functions of the first kind.
- It is possible to obtain another approximation for the asymptotic order parameter (than in the above mentioned papers) by means of the Lagrange's inversion theorem and also a rational approximation.

As far as we know the above mentioned subject was not studied yet in details from the mathematical point of view and we believe that the obtained results may be useful for the people working in statistical physics.

2. BOUNDS FOR THE ASYMPTOTIC ORDER PARAMETER

In this section our aim is to discuss, complement and extend the results from [BGP, SS] concerning bounds for the asymptotic order parameter of the stochastic Kuramoto model. Some new and recently discovered Turán type inequalities for modified Bessel functions of the first kind play an important role in this section. For more details on Turán type inequalities for modified Bessel functions of the first kind we refer to [B] and to the references therein.

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