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TWO UNIVERSALITY RESULTS FOR POLYNOMIAL REPRODUCING KERNELS

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Abstract. We prove two new universality results for polynomial reproducing kernels of compactly supported measures. The first applies to measures on the unit circle with a jump and a singularity in the weight at 1 and the second applies to area-type measures on a certain disconnected polynomial lemniscate. In both cases, we apply methods developed by Lubinsky to obtain our results.

Keywords: Orthogonal polynomials, universality, reproducing kernels, Christoffel functions, confluent hypergeometric functions

AMS Subject Classifications: Primary: 42C05, Secondary: 33C15, 46E22

1. Introduction

1.1. **Background and Results.** Given a finite, positive, and compactly supported measure μ with infinitely many points in its support, let $\{\varphi_n(z)\}_{n=0}^{\infty}$ be the corresponding sequence of orthonormal polynomials satisfying

$$\int \varphi_n(z)\overline{\varphi_m(z)}d\mu(z) = \delta_{m,n}.$$

The leading coefficient of φ_n is κ_n and φ_n/κ_n is a monic polynomial, which we denote by Φ_n. If ever it is necessary to specify the measure of orthogonality, we will write $\varphi_n(z;\mu)$, $\Phi_n(z;\mu)$, and $\kappa_n(\mu)$. The degree n polynomial reproducing kernel K_n is given by

$$K_n(z,w;\mu) := \sum_{m=0}^{n} \varphi_m(z)\overline{\varphi_m(w)}$$

and is so named because if $Q(z)$ is a polynomial of degree at most n, then

$$\int Q(z)K_n(w,z;\mu)d\mu(z) = Q(w).$$

When one speaks of universality limits for such kernels, one is interested in determining existence of the limit

$$\lim_{n\to\infty} \frac{K_n(z+\epsilon_1(n), z+\epsilon_2(n);\mu)}{K_n(z,z;\mu)}, \tag{1}$$

where $\epsilon_j(n) \to 0$ as $n \to \infty$ in a specific way for $j = 1,2$. The motivation for calculating such limits comes from random matrix theory and we refer the reader to [9, 25] for further details. The term "universality" is used when one can establish existence of the limit (1) for a large class of measures μ and points $z \in \mathrm{supp}(\mu)$ in such a way that the limiting