

Accepted Manuscript

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Brian Simanek

PII: S0021-9045(17)30002-3

DOI: <http://dx.doi.org/10.1016/j.jat.2017.01.002>

Reference: YJATH 5129

To appear in: *Journal of Approximation Theory*

Received date: 1 June 2016

Revised date: 31 December 2016

Accepted date: 8 January 2017



Please cite this article as: B. Simanek, Two universality results for polynomial reproducing kernels, *Journal of Approximation Theory* (2017), <http://dx.doi.org/10.1016/j.jat.2017.01.002>

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TWO UNIVERSALITY RESULTS FOR POLYNOMIAL REPRODUCING KERNELS

Brian Simanek

*Baylor University Department of Mathematics
One Bear Place #97328, Waco, TX 76798
Brian.Simanek@baylor.edu*

ABSTRACT. We prove two new universality results for polynomial reproducing kernels of compactly supported measures. The first applies to measures on the unit circle with a jump and a singularity in the weight at 1 and the second applies to area-type measures on a certain disconnected polynomial lemniscate. In both cases, we apply methods developed by Lubinsky to obtain our results.

Keywords: Orthogonal polynomials, universality, reproducing kernels, Christoffel functions, confluent hypergeometric functions

AMS Subject Classifications: Primary: 42C05, Secondary: 33C15, 46E22

1. INTRODUCTION

1.1. **Background and Results.** Given a finite, positive, and compactly supported measure μ with infinitely many points in its support, let $\{\varphi_n(z)\}_{n=0}^\infty$ be the corresponding sequence of orthonormal polynomials satisfying

$$\int \varphi_n(z) \overline{\varphi_m(z)} d\mu(z) = \delta_{m,n}.$$

The leading coefficient of φ_n is κ_n and φ_n/κ_n is a monic polynomial, which we denote by Φ_n . If ever it is necessary to specify the measure of orthogonality, we will write $\varphi_n(z; \mu)$, $\Phi_n(z; \mu)$, and $\kappa_n(\mu)$. The degree n polynomial reproducing kernel K_n is given by

$$K_n(z, w; \mu) := \sum_{m=0}^n \varphi_m(z) \overline{\varphi_m(w)}$$

and is so named because if $Q(z)$ is a polynomial of degree at most n , then

$$\int Q(z) K_n(w, z; \mu) d\mu(z) = Q(w).$$

When one speaks of universality limits for such kernels, one is interested in determining existence of the limit

$$\lim_{n \rightarrow \infty} \frac{K_n(z + \epsilon_1(n), z + \epsilon_2(n); \mu)}{K_n(z, z; \mu)}, \quad (1)$$

where $\epsilon_j(n) \rightarrow 0$ as $n \rightarrow \infty$ in a specific way for $j = 1, 2$. The motivation for calculating such limits comes from random matrix theory and we refer the reader to [9, 25] for further details. The term “universality” is used when one can establish existence of the limit (1) for a large class of measures μ and points $z \in \text{supp}(\mu)$ in such a way that the limiting

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