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Grzegorz Świderski

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PERIODIC PERTURBATIONS OF UNBOUNDED JACOBI MATRICES II: FORMULAS FOR DENSITY

GRZEGORZ ŚWIDERSKI

ABSTRACT. We give formulas for the density of the measure of orthogonality for orthonormal polynomials with unbounded recurrence coefficients. The formulas involve limits of appropriately scaled Turán determinants or Christoffel functions. Exact asymptotics of the polynomials and numerical examples are also provided.

1. INTRODUCTION

Consider a sequence $(p_n : n \in \mathbb{N})$ of polynomials defined by

$$(1) \qquad \begin{gathered} p_{-1}(x) = 0, \quad p_0(x) = 1, \\ a_{n-1}p_{n-1}(x) + b_n p_n(x) + a_n p_{n+1}(x) = x p_n(x) \quad (n \geq 0) \end{gathered}$$

for sequences $a = (a_n : n \in \mathbb{N})$ and $b = (b_n : n \in \mathbb{N})$ satisfying $a_n > 0$ and $b_n \in \mathbb{R}$. The sequence (1) is orthonormal in $L^2(\mu)$ for a Borel measure μ on the real line. We are interested in the case when the sequence a is unbounded and the measure μ is unique. When it holds, we want to find conditions on the sequences a and b assuring absolute continuity of μ and a constructive formula for its density.

In the case when the sequences a and b are bounded, there are several approaches to an approximation of the density of μ. One is obtained by means of *N-shifted Turán determinants*, i.e. expressions of the form

$$D_n^N(x) = p_n(x)p_{n+N-1}(x) - p_{n-1}(x)p_{n+N}(x)$$

for positive N (see [21, 10, 26]). Another by *Christoffel functions*, i.e.

$$\lambda_n(x) = \left[\sum_{k=0}^{n} p_k^2(x)\right]^{-1}$$

(see [20, 25]).

In the unbounded case there is a vast literature concerning qualitative properties of μ such as: its support, absolute continuity or continuity of μ, localization of its discrete part, see, e.g. [4, 6, 7, 14, 15, 16, 27]. As far as the approximation of μ is concerned, the only result known to the author is [1]. In Section 3 we prove the following theorem.

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