

## Accepted Manuscript

Periodic perturbations of unbounded Jacobi matrices II: Formulas for density

Grzegorz Świdorski

PII: S0021-9045(17)30004-7

DOI: <http://dx.doi.org/10.1016/j.jat.2017.01.004>

Reference: YJATH 5131

To appear in: *Journal of Approximation Theory*

Received date: 7 April 2016

Revised date: 7 November 2016

Accepted date: 3 January 2017

Please cite this article as: G. Świdorski, Periodic perturbations of unbounded Jacobi matrices II: Formulas for density, *Journal of Approximation Theory* (2017), <http://dx.doi.org/10.1016/j.jat.2017.01.004>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



# PERIODIC PERTURBATIONS OF UNBOUNDED JACOBI MATRICES II: FORMULAS FOR DENSITY

GRZEGORZ ŚWIDERSKI

**ABSTRACT.** We give formulas for the density of the measure of orthogonality for orthonormal polynomials with unbounded recurrence coefficients. The formulas involve limits of appropriately scaled Turán determinants or Christoffel functions. Exact asymptotics of the polynomials and numerical examples are also provided.

## 1. INTRODUCTION

Consider a sequence  $(p_n : n \in \mathbb{N})$  of polynomials defined by

$$(1) \quad \begin{aligned} p_{-1}(x) &= 0, \quad p_0(x) = 1, \\ a_{n-1}p_{n-1}(x) + b_np_n(x) + a_np_{n+1}(x) &= xp_n(x) \quad (n \geq 0) \end{aligned}$$

for sequences  $a = (a_n : n \in \mathbb{N})$  and  $b = (b_n : n \in \mathbb{N})$  satisfying  $a_n > 0$  and  $b_n \in \mathbb{R}$ . The sequence (1) is orthonormal in  $L^2(\mu)$  for a Borel measure  $\mu$  on the real line. We are interested in the case when the sequence  $a$  is unbounded and the measure  $\mu$  is unique. When it holds, we want to find conditions on the sequences  $a$  and  $b$  assuring absolute continuity of  $\mu$  and a constructive formula for its density.

In the case when the sequences  $a$  and  $b$  are bounded, there are several approaches to an approximation of the density of  $\mu$ . One is obtained by means of *N-shifted Turán determinants*, i.e. expressions of the form

$$D_n^N(x) = p_n(x)p_{n+N-1}(x) - p_{n-1}(x)p_{n+N}(x)$$

for positive  $N$  (see [21, 10, 26]). Another by *Christoffel functions*, i.e.

$$\lambda_n(x) = \left[ \sum_{k=0}^n p_k^2(x) \right]^{-1}$$

(see [20, 25]).

In the unbounded case there is a vast literature concerning qualitative properties of  $\mu$  such as: its support, absolute continuity or continuity of  $\mu$ , localization of its discrete part, see, e.g. [4, 6, 7, 14, 15, 16, 27]. As far as the approximation of  $\mu$  is concerned, the only result known to the author is [1]. In Section 3 we prove the following theorem.

---

2010 *Mathematics Subject Classification.* Primary: 42C05.

*Key words and phrases.* Continuous positive density, Turán determinants, Christoffel functions, asymptotics of orthonormal polynomials.

Download English Version:

<https://daneshyari.com/en/article/5773774>

Download Persian Version:

<https://daneshyari.com/article/5773774>

[Daneshyari.com](https://daneshyari.com)