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Equidistribution of Zeros of Random Polynomials

Igor Pritsker and Koushik Ramachandran

Abstract

We study the asymptotic distribution of zeros for the random polynomials $P_n(z) = \sum_{k=0}^{n} A_k B_k(z)$, where $\{A_k\}_{k=0}^{\infty}$ are non-trivial i.i.d. complex random variables. Polynomials $\{B_k\}_{k=0}^{\infty}$ are deterministic, and are selected from a standard basis such as Szegő, Bergman, or Faber polynomials associated with a Jordan domain G bounded by an analytic curve. We show that the zero counting measures of P_n converge almost surely to the equilibrium measure on the boundary of G if and only if $\mathbb{E}[\log^+ |A_0|] < \infty$.

1 Introduction

Zeros of polynomials of the form $P_n(z) = \sum_{k=0}^{n} A_k z^k$, where $\{A_n\}_{k=0}^{n}$ are random coefficients, have been studied by Bloch and Pólya, Littlewood and Offord, Erdős and Offord, Kac, Rice, Hammersley, Shparo and Shur, Arnold, and many other authors. The early history of the subject with numerous references is summarized in the books by Bharucha-Reid and Sambandham [10], and by Farahmand [12]. It is well known that, under mild conditions on the probability distribution of the coefficients, the majority of zeros of these polynomials accumulate near the unit circumference, being equidistributed in the angular sense. Introducing modern terminology, we call a collection of random polynomials $P_n(z) = \sum_{k=0}^{n} A_k z^k$, $n \in \mathbb{N}$, the ensemble of *Kac polynomials*. Let $\{Z_{k,n}\}_{k=1}^{n}$ be the zeros of a polynomial P_n of degree n, and define the *zero counting measure*

$$\tau_n = \frac{1}{n} \sum_{k=1}^{n} \delta_{Z_{k,n}}.$$

The fact of equidistribution for the zeros of random polynomials can now be expressed via the weak convergence of τ_n to the normalized arclength measure $\mu_{\mathbb{T}}$ on the unit circumference $\mathbb{T}$, where $d\mu_{\mathbb{T}}(e^{it}) := dt/(2\pi)$. Namely, we have that $\tau_n \xrightarrow{w} \mu_{\mathbb{T}}$ with probability 1 (abbreviated as a.s. or almost surely). More recent work on the global distribution of zeros of Kac polynomials include papers of Ibragimov and Zaporozhets [18], Kabluchko and Zaporozhets [19, 20], etc. In particular, Ibragimov and Zaporozhets [18] proved that if the coefficients are independent and identically distributed non-trivial random variables, then the condition $\mathbb{E}[\log^+ |A_0|] < \infty$ is necessary and sufficient for $\tau_n \xrightarrow{w} \mu_{\mathbb{T}}$ almost surely. Here, $\mathbb{E}[X]$ denotes the expectation of a random variable X, and X is called non-trivial if $\mathbb{P}(X = 0) < 1$.