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# An interpolation problem on the circle between Lagrange and Hermite problems

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**Abstract**

This paper is devoted to studying an interpolation problem on the circle, which can be considered an intermediate problem between Lagrange and Hermite interpolation. The difference as well as the novelty is that we prescribe Lagrange values at the $2n$ roots of a complex number with modulus one and we prescribe values for the first derivative only on half of the nodes. We obtain two types of expressions for the interpolation polynomials: the barycentric expressions and another one given in terms of an orthogonal basis of the corresponding subspace of Laurent polynomials. These expressions are very suitable for numerical computation. Moreover, we give sufficient conditions in order to obtain convergence in case of continuous functions and we obtain the rate of convergence for smooth functions. Finally we present some numerical experiments to highlight the results obtained.

*Keywords:* Interpolation; Laurent polynomials; Unit circle; Barycentric expression; Convergence; Rate of convergence.
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## 1. Introduction

Interpolation problems on the complex plane, in particular on the unit circle and on the real line, have been extensively studied for different situations, such as Lagrange and classical Hermite interpolation (see [22], [16], [20]), Birkhoff or Pál-type interpolation. An interesting approach is given in [5], where the authors present a unifying formulation of a number of related problems that require finding a non trivial linear combination of possible some of the values of a function and possibly some of its derivative values, at a number of data points. This type of problems include Lagrange, Hermite and Hermite-Birkhoff interpolation.

In 1975 Pál posed in [19] the following problem. Let $w$ be a polynomial of degree $n$ with simple real zeros $\{x_i\}_{i=1}^n$ and let $\{x_i^*\}_{i=1}^n$ be the zeros of $w'$. If $\{y_i\}_{i=1}^n$ and $\{y_i'\}_{i=1}^{n-1}$ are two systems of arbitrary real numbers, the problem is to determine a polynomial $P$ of the lowest possible degree satisfying the interpolation conditions $P(x_k) = y_k$, $k = 1, \cdots, n$ and $P'(x_k^*) = y_k'$, $k = 1, \cdots, n-1$.

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