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Exceptional Hahn and Jacobi orthogonal polynomials *

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Abstract

Using Casorati determinants of Hahn polynomials $(h_n^{\alpha,\beta,N})_n$, we construct for each pair $\mathcal{F}=(F_1,F_2)$ of finite sets of positive integers polynomials $h_n^{\alpha,\beta,N;\mathcal{F}}$, $n\in\sigma_{\mathcal{F}}$, which are eigenfunctions of a second order difference operator, where $\sigma_{\mathcal{F}}$ is certain set of nonnegative integers, $\sigma_{\mathcal{F}}\subsetneq\mathbb{N}$. When $N\in\mathbb{N}$ and α, β, N and $\mathcal{F}$ satisfy a suitable admissibility condition, we prove that the polynomials $h_n^{\alpha,\beta,N;\mathcal{F}}$ are also orthogonal and complete with respect to a positive measure (exceptional Hahn polynomials). By passing to the limit, we transform the Casorati determinant of Hahn polynomials into a Wronskian type determinant of Jacobi polynomials $(P_n^{\alpha,\beta})_n$. Under suitable conditions for α, β and $\mathcal{F}$, these Wronskian type determinants turn out to be exceptional Jacobi polynomials.

1 Introduction

In [8] and [9], we have introduced a systematic way of constructing exceptional discrete orthogonal polynomials using the concept of dual families of polynomials. We applied this procedure to construct exceptional Charlier and Meixner polynomials and, passing to the limit, exceptional Hermite and Laguerre polynomials, respectively. The purpose of this paper is to extend this construction to Hahn and Jacobi exceptional polynomials.

Exceptional orthogonal polynomials p_n, $n\in X\subsetneq\mathbb{N}$, are complete orthogonal polynomial systems with respect to a positive measure which in addition are eigenfunctions of a second order differential operator. They extend the classical families of Hermite, Laguerre and Jacobi. The last few years have seen a

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