Accepted Manuscript

Extended Traub-Woźniakowski convergence and complexity of Newton iteration in Banach space

I.K. Argyros, G.N. Silva

PII: S0885-064X(17)30065-1
DOI: http://dx.doi.org/10.1016/j.jco.2017.06.001
Reference: YJCOM 1335

To appear in: *Journal of Complexity*

Received date : 15 January 2017
Accepted date : 14 June 2017

Extended Traub-Woźniakowski convergence and complexity of Newton iteration in Banach space

I. K. Argyros * G. N. Silva †

June 16, 2017

Abstract

An optimal convergence condition for Newton iteration is presented which is at least as weak as the one obtained by Traub and Woźniakowski leading also to an at least as precise complexity. The novelty of the paper is the introduction of a restricted convergence domain. That is we find a more precise location where the Newton iterates lie than in earlier studies. Consequently the Lipschitz constants are at least as small as the ones used before. This way and under the same computational cost, we extend the local convergence as well as the complexity of Newton iteration. Numerical examples further justify the theoretical results.

Keywords: Newton iteration, Banach space, local convergence, complexity, restricted convergence domain.

Mathematical Subject Classification (2010): Primary 65K05; 65K15; 49M15; 49M37

1 Introduction

Many problems in computational sciences can be reduced to solving a nonlinear equation between two normed spaces using mathematical modeling. Exact solutions for such equations can be found only in some special cases. That explains why most solution methods for such equations are usually iterative. The study of the local convergence and the complexity of an iterative procedure is very important. That is finding how close the initial point should be to a solution to achieve high order convergence and how close the initial point has to be to the solution to guarantee good complexity. In particular, using our new idea of restricted convergence domains (to be precised in the next section) we revisit the Traub-Woźniakowski theory of convergence and complexity for the Newton iteration [18]. We show that an optimal convergence condition for the Newton iteration can be derived which is at least as weak as the previously obtained, leading also to an at least as precise complexity. These advantages are obtained under the same computational cost, since in practice the Lipschitz constants used are special cases of the constants used in [18]. The technique of restricted convergence domains can be used to study the local convergence and complexity for other iterative methods. In order to make the paper as self contained as possible, we restate some standard notations, results and definitions [1]–[18].

The remainder of this paper is structured as follows: Section 2 contains the local convergence of the Newton iteration; Section 3 the complexity and Section 4 the numerical examples.

*Cameron University, Department of Mathematics Sciences, Lawton, OK 73505, USA (Email: iargyros@cameron.edu).

†CCET/UFOB, CEP 47808-021 - Barreiras, BA, Brazil (Email: gilson.silva@ufob.edu.br). The author was supported in part by CAPES .