Accepted Manuscript

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PII: S0885-064X(16)30116-9
DOI: http://dx.doi.org/10.1016/j.jco.2016.12.002
Reference: YJCOM 1321

To appear in: *Journal of Complexity*

Received date: 7 January 2016
Accepted date: 19 December 2016

Please cite this article as: D. Amelunxen, M. Lotz, Average-case complexity without the black swans, *Journal of Complexity* (2016), http://dx.doi.org/10.1016/j.jco.2016.12.002

Average-case complexity without the black swans

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Abstract

We introduce the concept of weak average-case analysis as an attempt to achieve theoretical complexity results that are closer to practical experience than those resulting from traditional approaches. The underlying paradigm is accepted in other areas such as non-asymptotic random matrix theory and compressive sensing, and has a particularly convincing interpretation in the most common situation encountered for condition numbers, where it amounts to replacing a null set of ill-posed inputs by a "numerical null set". We illustrate the usefulness of these notions by considering three settings: (1) condition numbers that are inversely proportional to a distance of a homogeneous algebraic set of ill-posed inputs; (2) Renegar's condition number for conic optimization; (3) the running time of power iteration for computing a leading eigenvector of a Hermitian matrix.

Keywords: average-case analysis, smoothed analysis, condition numbers, power iteration, computational complexity, random matrix theory
2000 MSC: 68Q25, 68Q87, 60B20, 60H25, 90C25

1. Introduction

Depending on context and tradition, a computational problem can mean something practical that begs to be solved as efficiently as possible, or a mathematical object in its own right, to be analysed, classified, and understood. In the first sense, the aim is to develop methods that work well on problems of interest, while in the second, complexity-theoretic sense, algorithms are merely devices used to show that a problem can be solved within