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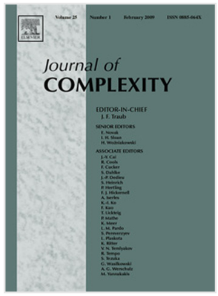

Stable splittings of Hilbert spaces of functions of infinitely many variables

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Stable splittings of Hilbert spaces of functions of infinitely many variables

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Abstract

We present an approach to defining Hilbert spaces of functions depending on infinitely many variables or parameters, with emphasis on a weighted tensor product construction based on stable space splittings. The construction has been used in an exemplary way for guiding dimension- and scale-adaptive algorithms in application areas such as statistical learning theory, reduced order modeling, and information-based complexity. We prove results on compact embeddings, norm equivalences, and the estimation of ϵ-dimensions. A new condition for the equivalence of weighted ANOVA and anchored norms is also given.

Keywords: Tensor product Hilbert spaces, weighted Hilbert space decompositions, functions of infinitely many variables, ϵ-dimensions, compact embeddings, L_2 approximation, high-dimensional model representation.
2000 MSC: 41A63, 41A45, 46E20, 46E22

1. Introduction

Functions depending on infinitely many real variables have been studied in different fields such as, e.g., stochastic processes, measure theory, stochastic and parametric PDEs, uncertainty quantification, and information-based complexity theory. A rich source of such functions is provided by nonlinear functionals $F : K \subset U \to \mathbb{R}$ defined on a subset of a separable Banach space U. Such F can often be parametrized by representing K in the form $K = \{\sum_{k\in\mathbb{N}} x_k u_k : x = (x_1, x_2, \ldots) \in \mathcal{X} \subset \mathbb{R}^\infty\}$, with $\{u_k\}$ an appropriately chosen generating system of elements in U. Then, $f(x) := F(\sum_{k\in\mathbb{N}} x_k u_k)$ is studied as a function of infinitely many variables over the domain $\mathcal{X} \subset \mathbb{R}^\infty$ instead of $K \subset U$.

To classify functions of infinitely many real variables and to quantify their properties, various function spaces have been introduced following the traditional constructions for

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