# Accepted Manuscript


On second order orthogonal Latin hypercube designs

Haralambos Evangelaras, Markos V. Koutras

PII: S0885-064X(16)30092-9
DOI: http://dx.doi.org/10.1016/j.jco.2016.10.007
Reference: YJCOM 1311

To appear in: *Journal of Complexity*

Received date: 7 March 2016
Accepted date: 26 October 2016

Please cite this article as: H. Evangelaras, M.V. Koutras, On second order orthogonal Latin hypercube designs, *Journal of Complexity* (2016), http://dx.doi.org/10.1016/j.jco.2016.10.007

# On second order orthogonal Latin hypercube designs

Haralambos Evangelaras*, Markos V. Koutras

*Department of Statistics and Insurance Science, University of Piraeus, Greece*

**Abstract**

In polynomial regression modeling, the use of a second order orthogonal Latin hypercube design guarantees that the estimates of the first order effects are uncorrelated to each other as well as to the estimates of the second order effects. In this paper, we prove that such designs with $n$ runs and $k > 2$ columns do not exist if $n \equiv 4 \bmod 8$. Furthermore, we prove that second order Latin hypercube designs with $k \geq 4$ columns that guarantee the orthogonality of two-factor interactions which do not share a common factor, do not exist for even $n$ that is not a multiple of 16. Finally, we investigate the class of symmetric orthogonal Latin hypercube designs (SOLHD), which are a special subset of second order orthogonal Latin hypercube designs. We describe construction techniques for SOLHDs with $n$ runs and (a) $k \leq 4$ columns, when $n \equiv 0 \bmod 8$ or $n \equiv 1 \bmod 8$, (b) $k \leq 8$ columns when $n \equiv 0 \bmod 16$ or $n \equiv 1 \bmod 16$ and (c) $k = 4$ columns when $n \equiv 0 \bmod 16$ or $n \equiv 1 \bmod 16$, that guarantee the orthogonality of two-factor interactions which do not share a common factor. Finally, we construct and enumerate all non-isomorphic SOLHDs with $n \leq 17$ runs and $k \geq 2$ columns, as well as with $19 \leq n \leq 20$ runs and $k = 2$ columns.

*Keywords:* Latin hypercube, Orthogonality, Polynomial models

## 1. Introduction

A Latin hypercube design $D(n, k)$ (McKay, Beckman and Conover (1979)) with $n$ runs and $k$ columns is an $n \times k$ matrix, in which each column is a random permutation of $1, 2, \ldots, n$. Therefore, the $j$-th column of a Latin hypercube design is of the form $(q_{1j}, q_{2j}, \ldots, q_{nj})^T$, where $q_{ij}$ are positive integers with $1 \leq q_{ij} \leq n$ and $q_{ij} \neq q_{kj}$ for $i \neq k$. This class of designs has been extensively exploited in computer experiments but they can also offer practical tools for screening experiments with quantitative factors having equally spaced levels. The need for resorting to Latin hypercube designs in all these circumstances, arises from the fact that it is desirable for the points of the design matrix to be uniformly scattered on the input space. When used under this framework, the most popular term for them in the related literature is space - filling or uniform designs. For a nice overview on computer experiments modeling, one may refer to Fang and Lin (2003), Santner et al. (2003) and Fang et al. (2006).

In the literature of Latin hypercube designs, great attention has been drawn to the so called Orthogonal Latin Hypercube Designs (OLHDs). In general, a Latin hypercube design $D(n, k)$ is said to be orthogonal, if all pairs of columns are uncorrelated; in other words, if $(q_{1j_1}, q_{2j_1}, \ldots, q_{nj_1})^T$ and $(q_{1j_2}, q_{2j_2}, \ldots, q_{nj_2})^T$ are two distinct columns of $D(n, k)$, then the next condition should hold true

$$\sum_{i=1}^{n} q_{ij_1} q_{ij_2} = (\sum_{i=1}^{n} q_{ij_1})(\sum_{i=1}^{n} q_{ij_2})/n = n(n+1)^2/4. \quad (1)$$

Note that, the second equality results from the fact that the elements of each column consist of a permutation of the first $n$ natural numbers. As Lin (2008) and Lin et al. (2010) have indicated, there exists no OLHD with $k > 1$ columns, for $n \equiv 2 \bmod 4$.

*Corresponding author

*Email addresses:* hevangel@unipi.gr (Haralambos Evangelaras), mkoutras@unipi.gr (Markos V. Koutras)