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# Nonlinear thermoelastic plate equations – Global existence and decay rates for the Cauchy problem

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**Abstract**

We consider the Cauchy problem in $\mathbb{R}^n$ for quasilinear thermoelastic Kirchhoff-type plate equations where the heat conduction is modeled by either the Cattaneo law or by the Fourier law. Additionally, we take into account possible inertial effects. Considering nonlinearities which are of fourth-order in the space variable, we deal with a quasilinear system which triggers difficulties typical for nonlinear Schrödinger equations. The different models considered are systems of mixed type comparable to Schrödinger–parabolic or Schrödinger–hyperbolic systems. The main task consists in proving sophisticated a priori estimates leading to obtaining the global existence of solutions for small data, neither known nor expected for the Cauchy problem in pure plate theory nor available before for the coupled system under investigation, where only special cases (bounded domains within with analytic semigroup setting, or the Cauchy problem with semilinear nonlinearities) had been treated before.


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*Keywords:* Quasilinear thermoelastic plate; Fourier and Cattaneo law; Global existence; Cauchy problem; Inertial term

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## 1. Introduction

We consider the Cauchy problem for the following nonlinear thermoplastic plate equation, where the heat conduction is modeled by Cattaneo's (Maxwell's, Vernotte's) law ($\tau > 0$) or by Fourier's law ($\tau = 0$), and where an inertial term may be present ($\mu > 0$) or not ($\mu = 0$):

$$\begin{aligned} u_{tt} + \Delta b(\Delta u) - \mu \Delta u_{tt} + \nu \Delta \theta &= 0, \\ \theta_t + \operatorname{div} q - \nu \Delta u_t &= 0, \\ \tau q_t + q + \nabla \theta &= 0. \end{aligned} \tag{1.1}$$

Here, $u$ describes the vertical displacement of a plate, while $\theta$ and $q$ denote the temperature (difference to a fixed temperature) and the heat flux, respectively. For the Cattaneo law, the relaxation parameter $\tau$ is a positive constant. The constant $\mu$ is a non-negative parameter in front of the inertial term. The function $b$ is a given smooth function which satisfies $b'(0) > 0$. Without loss of generality, we assume $b(0) = 0$. This is also physically sound to assume. Cf. reference [8].

Not affecting the mathematical aspects, we have set most physical constants usually appearing in the equations equal to one, just keeping the constant $\tau$, $\mu$ being relevant in particular for the type of the equations, and the positive $\nu$ for illustrating the coupling effect in the estimates.

Taking $\tau = 0$, we obtain the standard nonlinear thermoelastic plate equation:

$$\begin{aligned} u_{tt} + \Delta b(\Delta u) - \mu \Delta u_{tt} + \nu \Delta \theta &= 0, \\ \theta_t - \Delta \theta - \nu \Delta u_t &= 0, \end{aligned} \tag{1.2}$$

where the Cattaneo law

$$\tau q_t + q + \nabla \theta = 0 \tag{1.3}$$

has turned into the Fourier law

$$q + \nabla \theta = 0, \tag{1.4}$$

leading to the classical parabolic heat equation appearing in (1.2). Turning to neglecting variations in time of the temperature, i.e. assuming

$$\theta_t \equiv 0$$

in (1.2), the system reduced to the following damped nonlinear plate equation:

$$u_{tt} + \Delta b(\Delta u) - \mu \Delta u_{tt} - \nu^2 \Delta u_t = 0. \tag{1.5}$$

The purpose of this paper is to construct global solutions in time for the Cauchy problem to the equations (1.5), then for (1.2) ($\tau = 0$, with both cases $\mu > 0$ or $\mu = 0$), and finally (1.1) with $\tau > 0$ and $\mu > 0$. Simultaneously we will describe the asymptotic behavior of the global solutions.

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