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Optimal decay rate for the compressible Navier–Stokes–Poisson system in the critical L^p framework ☆

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Abstract

In this paper, we consider the large time behavior of global solutions to the initial value problem for the compressible Navier–Stokes–Poisson system *in the* L^p *critical framework and in any dimension* $N \geq 3$. We obtain the time decay rates, not only for Lebesgue spaces, but also for a family of Besov norms *with negative or nonnegative regularity exponents*, which improves the decay results in high Sobolev regularity. The proof is mainly based on the Littlewood–Paley theory and refined time weighted inequalities in Fourier space.

MSC: 35Q35; 35B40; 76N15

Keywords: Time-decay rates; Navier–Stokes–Poisson equations; Critical spaces; L^p framework

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1. Introduction

In this paper, we investigate the long-time behavior of global strong solutions for the following compressible Navier–Stokes–Poisson equations (NSP) in $\mathbb{R}^+ \times \mathbb{R}^N$ $(N \geq 3)$:

$$\begin{cases} \partial_t \rho + \mathrm{div}(\rho \mathbf{u}) = 0, \\ \partial_t(\rho \mathbf{u}) + \mathrm{div}(\rho \mathbf{u} \otimes \mathbf{u}) - \mu \Delta \mathbf{u} - (\mu + \lambda)\nabla \mathrm{div}\mathbf{u} + \nabla P(\rho) = \rho \nabla \Phi, \\ \Delta \Phi = \rho - \bar{\rho}, \\ (\rho, \mathbf{u})(0) = (\rho_0, \mathbf{u}_0), \end{cases} \tag{1.1}$$

where ρ, $\mathbf{u}$ and Φ represent the electron density, the electron velocity and the electrostatic potential, respectively. The pressure P is a smooth function of ρ with $P'(\rho) > 0$ for $\rho > 0$, and the viscosity coefficients μ, λ are constants and satisfy $\mu > 0$ and $\nu := \lambda + 2\mu > 0$. Such a condition ensures ellipticity for the operator $\mu\Delta + (\lambda + \mu)\nabla \mathrm{div}$ and is satisfied in the physical cases. $\bar{\rho} > 0$ describes the background doping profile, and in this paper, for simplicity, we set $\bar{\rho} = 1$ and suppose that $P'(1) = 1$. The compressible NSP system could be used to model and simulate the transportation of charged particles in semiconductor devices [26].

The main purpose of this paper is to investigate the time decay rates of strong solutions to system (1.1) in the critical L^p framework. Here we observe that system (1.1) is invariant by the transformation

$$\tilde{\rho} = \rho(l^2 t, lx), \quad \tilde{\mathbf{u}} = l\mathbf{u}(l^2 t, lx)$$

up to a change of the pressure law $\tilde{P} = l^2 P$. A critical space is a space in which the norm is invariant under the scaling $(\tilde{e}, \tilde{\mathbf{f}})(x) = (e(lx), l\mathbf{f}(lx))$.

As regarding the existence of solutions to the compressible NSP equations, there are many important progresses. In terms of the local and global weak solutions, one can refer to [10–12,37] and references therein. In [32], Tan and Wang considered the global existence of weak solutions to the compressible magnetohydrodynamics with the Poisson term of Coulomb force in two dimensions. The global existence of small strong solutions to the compressible NSP equations in H^N Sobolev spaces was shown by Li, Matsumura and Zhang [23] in $\mathbb{R}^3$, while global existence of small solutions in the critical L^2 type hybrid Besov spaces in $\mathbb{R}^N$ $(N \geq 3)$ was obtained in [14]. Later on, Zheng [38] extended the result of [14] to the critical L^p framework (see Theorem 2.1 below).

One may wonder how the global solutions established in the critical L^p framework behave for large time. To make a clearer introduction to our result, we will recall some known convergence results for the compressible Navier–Stokes and NSP equations, respectively.

For the compressible Navier–Stokes equations, the first achievement is due to Matsumura and Nishida [27,28]. There, they proved the global existence of classical solutions for the initial perturbation small in $(L^1 \cap H^3)(\mathbb{R}^3)$ and established the following decay estimate:

$$\|(\rho - 1, \mathbf{u})(t)\|_{L^2} \leq C(1 + t)^{-\frac{3}{4}}. \tag{1.2}$$

This is the same as for the heat equation with data in $L^1(\mathbb{R}^3)$ and it turns out to be the optimal one for the corresponding linearized system. As a consequence, it is often referred to as the optimal time-decay rate.

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