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Radial singular solutions for a fourth order equation with negative exponents ☆

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Abstract

In this paper, we consider the radial singular solution of

$$\Delta^2 u = -\frac{1}{u^p}, \quad \text{in } \mathbb{R}^N \text{ with } u > 0,\, p > 0.$$

By using some elementary ordinary differential equation arguments, we prove the above equation admits no radial singular solution for $N = 3$, $p \geq 3$. In addition, the exact asymptotic behaviors of the radial singular solution as $r \to 0$ is established for $p = 1, N \geq 4$, which have a significant difference with the explicit singular solution. As a product, the singularity of any radial singular solution is of *type II* if $p = 1$, $N \geq 4$.

Keywords: Biharmonic equation; Singularity; Asymptotic behavior

1. Introduction

In this paper, we consider the following fourth order elliptic problems with negative exponents

$$\Delta^2 u = -\frac{1}{u^p}, \quad \text{in } \mathbb{R}^N \text{ with } u > 0,\, p > 0. \tag{1.1}$$

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The motivation to study this equation arises from the conformal geometry. It is well known that if g and $\bar{g}$ are two metrics on a 2-dimension Riemann surface and $\bar{g} = e^{2u} g$, then their associated Laplace operators satisfy

$$\Delta_{\bar{g}} = e^{-2u} \Delta_g,$$

where $\Delta_g = -\mathrm{div}_g \nabla$ is the Laplace operator with respect to g. Such an operator is known as conformally covariant operator. However, the Laplace operator Δ_g is neither conformally invariant nor conformally covariant for dimension $N \geq 3$, thus one usually defines the conformal Laplace in dimension $N \geq 3$ as

$$L = -\Delta_g + \frac{N-2}{4(N-1)} S_g,$$

which is a generalization of Laplace operator on a 2-dimension Riemann surface. Here S_g is the scalar curvature of the metric g.

Similarly Paneitz in [1] extended the Δ_g in dimension 2 to the fourth order operator on a 4-dimensional Riemannian manifold (M, g), which is defined as

$$P_g^4 = \Delta_g^2 - \mathrm{div}_g \Big(\frac{2}{3} S_g g - 2Ric_g \Big) d, \tag{1.2}$$

where Ric_g is the Ricci curvature of g. Also the P_g^4 has the conformally covariant property, more specifically, if $\tilde{g} = e^{2u} g$ for all $u \in C^\infty(M)$, then

$$P_{\tilde{g}}^4 = e^{-4u} P_g^4.$$

An extension to manifolds of other dimension, due to Branson [2], is the fourth order operator defined by

$$P_g^N = \Delta_g^2 - \mathrm{div}_g \Big(\frac{(N-2)^2 + 4}{2(N-1)(N-2)} S_g g - \frac{4}{N-2} Ric_g \Big) d + \frac{N-4}{2} Q_g^N, \tag{1.3}$$

where

$$Q_g^N = \frac{1}{2(N-1)} \Delta_g S_g + \frac{N^3 - 4N^2 + 16N - 16}{8(N-1)^2(N-2)^2} S_g^2 - \frac{2}{(N-2)^2} |Ric_g|^2.$$

We may note that when $N = 4$, Eq. (1.3) reduce to Eq. (1.2).

Similar to the conformal Laplace, P_g^N has conformal properties: for all $u \in C^\infty(M)$, $P_g^N(u\varphi) = \varphi^{\frac{N+4}{N-4}} P_{\tilde{g}}^N(u)$ when $\tilde{g} = \varphi^{\frac{4}{N-4}} g$. In particular, if $\varphi = 1$, then u satisfies the fourth-order Yamabe's equation

$$P_g^N u = \frac{N-4}{2} Q_{\tilde{g}} u^{\frac{N+4}{N-4}} \tag{1.4}$$