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Pulsating flows of the 2D Euler–Poisson equations

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Abstract

It is well known that solutions of gaseous or fluid dynamical systems can easily blowup or develop shock waves in finite time.

In this paper we show the existence of a class of "radially symmetric" rotational solutions to the two-dimensional pressureless Euler–Poisson equations. The flows are global (i.e., exist for all $t > 0$), have compact support at all times, and pulsate periodically. The method of construction is novel. It comprises the piecing together of suitable shells of moving particles in a delicate manner with careful choice of initial data. Each shell is the solution of a member of a continuum of ordinary differential equations. Detailed analysis of the equations is carried out to ensure that neighboring shells can be chosen to pulsate with the same period. We also show that any such solution can be extended to a larger flow by adding annuli of pulsating flows. Our result exhibits another example of rotation preventing the blowup of solutions.

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Keywords: Euler–Poisson equations; Rotating fluids; Pulsating; Periodic solutions; Pressureless fluids

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1. Introduction

The N-dimensional compressible Euler–Poisson equations can be written in the following form:

$$\begin{cases} \rho_t + \nabla \cdot (\rho \vec{u}) &= 0, \\ \rho \left[\vec{u}_t + (\vec{u} \cdot \nabla) \vec{u} \right] + \nabla P &= \rho \nabla \Phi, \\ \Delta \Phi(\vec{x}, t) &= -\alpha(N) \rho, \end{cases} \tag{1.1}$$

where $\alpha(N)$ is a constant related to the surface area of the unit ball in R^N: $\alpha(1) = 1$, $\alpha(2) = 2\pi$ and $\alpha(3) = 4\pi$, etc., and $P = P(\rho)$ is the pressure function. The unknown functions $\rho = \rho(\vec{x}, t)$, $\vec{u} = \vec{u}(\vec{x}, t) = (u_1, u_2, ..., u_N) \in \mathbf{R}^N$, and $\Phi(\vec{x}, t)$ are the density, the velocity and the self-gravitational potential field, respectively. The system is said to have an attractive force (between the fluid particles) because the right-hand side of $(1.1)_2$ is positive. If, instead, a minus side is added to the right-hand side, the system is said to have a repulsive force. For $N = 3$, the equations (1.1) has been used as the classical (non-relativistic) model of a galaxy in astrophysics. See [2] and [4], for some background information on this subject area.

Existence and stability results can be found, for example, in [1], [3], [6], [9], [11], [14–16], [19–23] and [25]. For the construction of special analytical solutions, see [4], [7], [13], [20], [24], [27] and [28].

For the most part, we are concerned with classical solutions with compact support density functions. More specifically, for each t, $\rho(\vec{x}, t)$ is assumed to have continuous first derivatives and vanishes outside a ball of finite radius, say, R. The velocity function $\vec{u}(\vec{x}, t)$, on the other hand, is only assumed to be C^1 and not required to have compact support. Local existence results guarantee that a C^1 solution exists in some time interval $[0, t^*)$, $t^* > 0$. Since the system is hyperbolic, if the fluid initially (i.e. at $t = 0$) has compact support, it will continue to have compact support at any fixed future time $t > 0$ as long as the solution exists. However, the support may grow, and may even grow out of bound, with time. If t^* can be chosen as large as we please, the solution is said to be global. In the contrary case, there is an upper bound of all such t^*. Without loss of generality, we may assume that t^* has already been chosen to be this upper bound. As $t \to t^*-$, the regularity of the solution is lost, either due to a blowup of ρ or $\vec{u}$ at some finite point $\vec{x}_0$, or a blowup of one of their first derivatives. In the rest of the paper, the term "blowup" refers to either of these two cases.

One of the well known examples of a break-down of solution regularity is the development of shock waves in solutions of the Burger's equation. The onset of turbulence in fluid motion is another. As a general rule, blowup solutions are more interesting and more difficult to study than global ones. Recently, in the case of one-dimensional or radially symmetric irrotational higher-dimensional flows, Engelberg, Liu, and Tadmor [8] and Liu and Tadmor [18] introduce the concept of critical thresholds. They discover that, in many cases, certain inequality involving the initial state of a flow determines whether finite-time blowup occurs later or not. Intuitively speaking, if initially the flow is quickly expanding, meaning that the flow particles all move away from each other with relative velocity exceeding a certain threshold, then the gravitational force will not be strong enough to pull the particles towards each other so as to cause collision in finite time. Compare this with our expanding universe.

Chae and Tadmor in [3] study the blowup phenomenon of the pressureless Euler–Poisson system (meaning that the term in ∇P $(1.1)_2$ is assumed to be 0) with attractive forces. Such systems