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Global well-posedness of classical solutions to a fluid–particle interaction model in $\mathbb{R}^3$

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Abstract

We consider the Cauchy problem of a fluid–particle interaction model in $\mathbb{R}^3$, namely, compressible Navier–Stokes equations coupled with Smoluchowski equation. When the initial data (ρ_0, u_0, η_0) is of small energy around steady state $(\rho_\star, 0, \eta_\star)$, the global well-posedness and large-time behavior of classical solutions are investigated. Vacuum is allowed.

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1. Introduction

The form of a fluid–particle interaction model called as Navier–Stokes–Smoluchowski equations in [2,5,6] is governed by the Smoluchowski equation coupled with the Navier–Stokes equations for a compressible fluid in $\mathbb{R}^3$ as follows:

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$$\rho_t + \nabla \cdot (\rho u) = 0, \tag{1.1}$$

$$\rho u_t + \rho u \cdot \nabla u + \nabla(p_F + \eta) - \mu \Delta u - \lambda \nabla(\nabla \cdot u) = -(\eta + \beta\rho)\nabla\Phi, \tag{1.2}$$

$$\eta_t + \nabla \cdot (\eta(u - \nabla\Phi)) = \Delta\eta, \tag{1.3}$$

where the density of the fluid $\rho \geq 0$, the fluid velocity field $u = (u^1, u^2, u^3)$, and the density of the particles in the mixture $\eta \geq 0$ is related to the probability distribution function $f(t, x, \xi)$ in the macroscopic description through the relation

$$\eta(t, x) = \int_{\mathbb{R}^3} f(t, x, \xi) d\xi.$$

Here $\nabla \cdot (= \text{div})$ denotes the spatial divergence operator on $\mathbb{R}^3$, the function p_F denotes the pressure of the fluid, given by

$$p_F = p_F(\rho) = a\rho^\gamma, \quad a > 0, \quad \gamma > 1. \tag{1.4}$$

And the time independent external potential $\Phi = \Phi(x) : \mathbb{R}^3 \to \mathbb{R}_+$ is the effects of gravity and buoyancy, β is a constant reflecting the differences in how the external force affects the fluid and the particles, λ and μ are constant viscosity coefficients satisfying the physical condition:

$$\mu > 0, \quad \lambda + \frac{2}{3}\mu \geq 0. \tag{1.5}$$

Without the dynamic viscosity terms in (1.2), this system was derived by Carrillo and Goudon [5], in which they introduced the flowing regime and the bubbling regime with respect to two different scalings and investigated the stability and asymptotic limits. The system (1.1)–(1.3) is known as the bubbling regime. When the viscous effects are taken into account, there are some previously relevant works on the system (1.1)–(1.3). For the global existence of weakly dissipative solutions as well as their weak-strong uniqueness and low Mach number limits in high dimensions, please refer to Ballew–Trivisa's work [3], Carrillo et al.'s work [6] and Ballew's work [1], respectively. In particular, Carrillo et al. in their work [6] proved that the weak solutions exist globally in time and that the weak solutions converge to a stationary solution as time goes to ∞.

When η is zero, the system (1.1)–(1.3) is reduced to compressible Navier–Stokes equations for isentropic flow. Even for the compressible Navier–Stokes equations for isentropic flow, the uniqueness of global weak solutions derived by P.L. Lions (refer to [17]. See also [9,14]) is still open. Thus, it is natural to study some regular solutions that can ensure the uniqueness. Refer to [2,11] for the local well-posedness of strong solutions in a bounded domain and classical solutions in $\mathbb{R}^3$, respectively. In one dimension, the global well-posedness of classical solutions with large initial data and vacuum was derived by Fang et al. [8]. In three dimensions, Chen, the first author, and Wang [7] established the existence (also uniqueness) theory of global classical solutions under some smallness assumptions on both the external potential and the perturbation of the initial data in a neighborhood of a stationary profile $(\rho_*, 0, 0)$ in some Sobolev spaces. The method in their proofs in [7] relies on the analysis of the corresponding linearized system and the time-decay estimates of η in L^2. Thus, that the initial density ρ_0 has positive lower bound is quite essential, i.e., no vacuum at any point.