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Strichartz estimates for the fractional Schrödinger and wave equations on compact manifolds without boundary

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Abstract

We firstly prove Strichartz estimates for the fractional Schrödinger equations on $\mathbb{R}^d$, $d \geq 1$ endowed with a smooth bounded metric g. We then prove Strichartz estimates for the fractional Schrödinger and wave equations on compact Riemannian manifolds without boundary (M, g). This result extends the well-known Strichartz estimate for the Schrödinger equation given in [1]. We finally give applications of Strichartz estimates for the local well-posedness of the pure power-type nonlinear fractional Schrödinger and wave equations posed on (M, g).

MSC: 35A01; 35A17; 35B45; 35Q55

Keywords: Nonlinear fractional Schrödinger equation; Strichartz estimates; WKB approximation; Pseudo-differential calculus

1. Introduction and main results

This paper is concerned with the Strichartz estimates for the generalized fractional Schrödinger equation on Riemannian manifold (M, g), namely

$$\begin{cases} i\partial_t u + \Lambda_g^\sigma u &= 0, \\ u(0) &= u_0, \end{cases}$$

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where $\sigma \in (0,\infty)\backslash\{1\}$ and $\Lambda_g = \sqrt{-\Delta_g}$ with Δ_g is the Laplace–Beltrami operator associated to the metric g. When $\sigma \in (0,2)\backslash\{1\}$, it corresponds to the fractional Schrödinger equation discovered by N. Laskin (see [2,3]). When $\sigma \geq 2$, it can be seen as a generalization of the Schrödinger equation $\sigma = 2$ (see e.g. [4,5]) or the fourth-order Schrödinger equation $\sigma = 4$ (see e.g. [6,7]).

The Strichartz estimates play an important role in the study of nonlinear fractional Schrödinger equation on $\mathbb{R}^d$ (see e.g. [4–6,8–12] and references therein). Let us recall the local in time Strichartz estimates for the fractional Schrödinger operator on $\mathbb{R}^d$. For $\sigma \in (0,\infty)\backslash\{1\}$ and $I \subset \mathbb{R}$ a bounded interval, one has

$$\|e^{it\Lambda^\sigma}u_0\|_{L^p(I,L^q(\mathbb{R}^d))} \leq C\|u_0\|_{H^{\gamma_{pq}}(\mathbb{R}^d)}, \tag{1.1}$$

where $\Lambda = \sqrt{-\Delta}$ with Δ is the free Laplace operator on $\mathbb{R}^d$ and

$$\gamma_{pq} = \frac{d}{2} - \frac{d}{q} - \frac{\sigma}{p}$$

provided that (p,q) satisfies the fractional admissible condition, namely

$$p \in [2,\infty], \quad q \in [2,\infty), \quad (p,q,d) \neq (2,\infty,2), \quad \frac{2}{p} + \frac{d}{q} \leq \frac{d}{2}.$$

We refer to [12] (see also [10]) for a general version of these Strichartz estimates on $\mathbb{R}^d$.

The main purpose of this paper is to prove Strichartz estimates for the fractional Schrödinger equation on $\mathbb{R}^d$ equipped with a smooth bounded metric and on a compact manifold without boundary (M,g).

Let us firstly consider $\mathbb{R}^d$ endowed with a smooth Riemannian metric g. Let $g(x) = (g_{jk}(x))_{j,k=1}^d$ be a metric on $\mathbb{R}^d$, and denote $G(x) = (g^{jk}(x))_{j,k=1}^d := g^{-1}(x)$. The Laplace–Beltrami operator associated to g reads

$$\Delta_g = \sum_{j,k=1}^d |g(x)|^{-1}\partial_j\left(g^{jk}(x)|g(x)|\partial_k\right),$$

where $|g(x)| := \sqrt{\det g(x)}$ and denote $P := -\Delta_g$ the self-adjoint realization of $-\Delta_g$. Recall that the principal symbol of P is

$$p(x,\xi) = \xi^t G(x)\xi = \sum_{j,k=1}^d g^{jk}(x)\xi_j\xi_k.$$

In this paper, we assume that g satisfies the following assumptions.

1. There exists $C > 0$ such that for all $x, \xi \in \mathbb{R}^d$,

$$C^{-1}|\xi|^2 \leq \sum_{j,k=1}^d g^{jk}(x)\xi_j\xi_k \leq C|\xi|^2. \tag{1.2}$$