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Inverse nodal problems for Dirac-type integro-differential operators

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Abstract

The inverse nodal problem for Dirac differential operator perturbated by a Volterra integral operator is studied. We prove that dense subset of the nodal points determines the coefficients of differential part and gives partial information on the coefficients of integral part of the operator. We also provide an algorithm to reconstruct the coefficients of the problem by using the nodal points.

MSC: 34A55; 34L05; 34K29; 34K10; 47G20

Keywords: Dirac operator; Integral-differential operators; Inverse nodal problem; Uniqueness theorem; Spectral problems

1. Introduction

We consider the boundary value problem L generated by the system of Dirac integro-differential equations:

$$\ell[Y(x)] := BY'(x) + \Omega(x)Y(x) + \int_0^x M(x,t)Y(t)dt = \lambda Y(x), \quad x \in (0,\pi), \tag{1}$$

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subject to the boundary conditions

$$U(y) := y_1(0)\sin\alpha + y_2(0)\cos\alpha = 0 \tag{2}$$

$$V(y) := y_1(\pi)\sin\beta + y_2(\pi)\cos\beta = 0 \tag{3}$$

where α, β are real constants and λ is the spectral parameter, $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\Omega(x) = \begin{pmatrix} V(x)+m & 0 \\ 0 & V(x)-m \end{pmatrix}$, $M(x,t) = \begin{pmatrix} M_{11}(x,t) & M_{12}(x,t) \\ M_{21}(x,t) & M_{22}(x,t) \end{pmatrix}$, $Y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$, $\Omega(x)$ and $M(x,t)$ are real-valued functions in the class of $W_2^1(0,\pi)$, where m is a real constant. Throughout this paper, we denote $p(x) = V(x)+m$, $r(x) = V(x)-m$.

In 1988, the first results of the inverse nodal Sturm–Liouville problem was given by McLaughlin [14] who proved that the potential of the Sturm–Liouville problem can be determined by a given dense subset of nodal points of the eigenfunctions. In 1989, Hald and McLaughlin consider more general boundary conditions and give some numerical schemes for the reconstruction of the potential from nodal points [10]. Yang provided an algorithm to solve inverse nodal Sturm–Liouville problem in 1997 [18]. Inverse nodal problems for Sturm–Liouville or diffusion operators have been studied in the several papers ([2], [3], [4], [7], [15], [16], [17], [20] and [21]). The inverse nodal problems for Dirac operators with various boundary conditions have been solved in [9], [19] and [22]. In their works, it was shown that the zeros of the first components of the eigenfunctions determines the coefficients of operator.

Nowadays, the studies concerning the perturbation of a differential operator by a Volterra integral operator, namely the integro-differential operator, are beginning to have a significant place in the literature ([5], [6], [8], [12] and [13]). For Sturm–Liouville type integro-differential operators, there exist some studies about inverse problems. However, there is only one study for Dirac type integro-differential operators [1] and our paper is the first result in inverse nodal problems for Dirac type integro-differential operators. The inverse nodal problem for Sturm–Liouville type integro-differential operators was first studied by [11]. In their study, it is shown that the potential function can be determined by using nodal points while the coefficient of the integral operator is known. In our study, we prove that the integral operator can be partially determined as well as the potential function and the other coefficients of the problem.

2. Preliminaries

Let $\varphi(x,\lambda) = (\varphi_1(x,\lambda), \varphi_2(x,\lambda))^T$ be the solution of (1) satisfying the initial condition $\varphi(0,\lambda) = (\cos\alpha, -\sin\alpha)^T$. For each fixed x and t, this solution is an entire function of λ.

It is clear that $\varphi(x,\lambda)$ satisfies the following integral equations:

$$\varphi_1(x,\lambda) = \cos(\lambda x - \alpha) + \int_0^x \sin\lambda(x-t)p(t)\varphi_1(t,\lambda)dt$$

$$+ \int_0^x \cos\lambda(x-t)r(t)\varphi_2(t,\lambda)dt \tag{4}$$