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Almost periodic solutions for an asymmetric oscillation

Peng Huang [a], Xiong Li [a,*,1], Bin Liu [b,2]

[a] School of Mathematics Sciences, Beijing Normal University, Beijing 100875, PR China
[b] School of Mathematical Sciences, Peking University, Beijing 100871, PR China

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Abstract

In this paper we study the dynamical behavior of the differential equation

$$x'' + ax^+ - bx^- = f(t),$$

where $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$, a and b are two different positive constants, $f(t)$ is a real analytic almost periodic function. For this purpose, firstly, we have to establish some variants of the invariant curve theorem of planar almost periodic mappings, which was proved recently by the authors (see [11]). Then we will discuss the existence of almost periodic solutions and the boundedness of all solutions for the above asymmetric oscillation.

Keywords: Almost periodic mappings; Invariant curve theorem; Asymmetric oscillations; Almost periodic solutions; Boundedness

1. Introduction

In this paper we continue the work initiated in [17] trying to understand the dynamic behavior of an asymmetric oscillation

* Corresponding author.
E-mail addresses: hp@mail.bnu.edu.cn (P. Huang), xli@bnu.edu.cn (X. Li), bliu@pku.edu.cn (B. Liu).

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$$x'' + ax^+ - bx^- = f(t), \tag{1.1}$$

where a, b are two different positive constants, $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$, $f(t)$ is a real analytic almost periodic function with the frequency $\omega = (\cdots, \omega_\lambda, \cdots)_{\lambda\in\mathbb{Z}} \in \mathbb{R}^{\mathbb{Z}}$, and $\omega = (\cdots, \omega_\lambda, \cdots)_{\lambda\in\mathbb{Z}}$ is a bilateral infinite sequence of rationally independent frequency, that is to say, any finite segments of ω are rationally independent. The general question that we have in mind is: under what conditions on a, b, ω and $f(t)$, (1.1) has almost periodic solutions and all solutions are bounded?

Due to the relevance with applied mechanics, for example, modeling some kind of suspension bridge (see [13]), the following semilinear Duffing's equation

$$x'' + ax^+ - bx^- = f(x, t) \tag{1.2}$$

was widely studied, where $f(x, t)$ is a smooth 2π-periodic function in t.

If the function $f(x, t)$ in (1.2) depends only on the time t, equation (1.2) becomes (1.1), which had been studied by Dancer [2] and Fučik [4] in their investigations of boundary value problems associated to equations with jumping nonlinearities. For recent development, we refer to [5,8,12] and references therein.

Ortega [17] investigated the Lagrangian stability for the equation

$$x'' + ax^+ - bx^- = 1 + \gamma p(t), \tag{1.3}$$

where γ is a small parameter. He proved that if $|\gamma|$ is sufficiently small and $p \in \mathcal{C}^4(\mathbb{S}^1)$ with $\mathbb{S}^1 = \mathbb{R}/2\pi\mathbb{Z}$, then all solutions are bounded, that is, for every solution $x(t)$, it is defined for all $t \in \mathbb{R}$ and

$$\sup_{t\in\mathbb{R}}(|x(t)| + |x'(t)|) < +\infty.$$

On the other hand, when

$$\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} \in \mathbb{Q}, \tag{1.4}$$

Alonso and Ortega [1] constructed a 2π-periodic function $f(t)$ such that all solutions of equation (1.1) with large initial conditions are unbounded. Moreover, for such a function $f(t)$, equation (1.1) has periodic solutions. This means that the unbounded solutions and periodic solutions coexist.

Liu [14] removed the smallness assumption on $|\gamma|$ in equation (1.3), and proved the boundedness of all solutions of equation (1.1) under the resonant condition (1.4) and some other reasonable assumptions.

Liu [15] dealt with the existence of quasi-periodic solutions and the boundedness of all solutions of equation (1.1) when $f(t)$ is a real analytic, even and quasi-periodic function with the frequency $\omega = (\omega_1, \omega_2, \cdots, \omega_n)$. Firstly, he obtained some invariant curve theorems of planar reversible mappings which are quasi-periodic in the spatial variable. As an application, he used the invariant curve theorem to investigate the existence of quasi-periodic solutions and the boundedness of all solutions of (1.1).

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