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Existence and asymptotic behavior of vector solutions for coupled nonlinear Kirchhoff-type systems

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Abstract

This paper deals with the following linearly coupled nonlinear Kirchhoff-type system:

$$\begin{cases} -\Big(a_1 + b_1 \int\limits_{\mathbb{R}^3} |\nabla u|^2 \,\mathrm{d}x\Big)\Delta u + \mu_1 u = f(u) + \beta v & \text{in } \mathbb{R}^3, \\ -\Big(a_2 + b_2 \int\limits_{\mathbb{R}^3} |\nabla v|^2 \,\mathrm{d}x\Big)\Delta v + \mu_2 v = g(v) + \beta u & \text{in } \mathbb{R}^3, \\ u, v \in H^1(\mathbb{R}^3), \end{cases}$$

where $a_i > 0, b_i \geq 0, \mu_i > 0$ are constants for $i = 1, 2$, $\beta > 0$ is a parameter and $f, g \in C(\mathbb{R}, \mathbb{R})$. Under the general Berestycki–Lions type assumptions on f and g, we establish the existence of positive vector solutions and positive vector ground state solutions respectively by using variational methods. We also study the asymptotic behavior of these solutions as $\beta \to 0^+$.

MSC: 35J50; 35A01; 35B40

Keywords: Kirchhoff-type system; General nonlinearity; Positive vector solution; Asymptotic behavior; Pohozaev identity; Variational method

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1. Introduction and main results

In this paper we study the existence and asymptotic behavior of positive vector solutions for the following linearly coupled nonlinear Kirchhoff-type system:

$$\begin{cases} -\Big(a_1 + b_1 \int\limits_{\mathbb{R}^3} |\nabla u|^2 \,\mathrm{d}x\Big)\Delta u + \mu_1 u = f(u) + \beta v \quad \text{in } \mathbb{R}^3, \\ -\Big(a_2 + b_2 \int\limits_{\mathbb{R}^3} |\nabla v|^2 \,\mathrm{d}x\Big)\Delta v + \mu_2 v = g(v) + \beta u \quad \text{in } \mathbb{R}^3, \\ u, v \in H^1(\mathbb{R}^3), \end{cases} \tag{1.1}$$

where $a_i > 0, b_i \geq 0, \mu_i > 0$ are constants for $i = 1, 2$, $\beta > 0$ is a coupling parameter. Our assumptions on f and g are as follows:
(A_1) $f, g \in C(\mathbb{R}, \mathbb{R})$, $\lim\limits_{t\to 0^+} \frac{f(t)}{t} = 0 = \lim\limits_{t\to 0^+} \frac{g(t)}{t}$ and $f(t), g(t) \equiv 0$ for all $t \leq 0$;
(A_2) there exist $p, q \in (2, 6)$ such that

$$\lim_{|t|\to+\infty} \frac{f(t)}{|t|^{p-1}} = 0 \quad \text{and} \quad \lim_{|t|\to+\infty} \frac{g(t)}{|t|^{q-1}} = 0;$$

(A_3) there exist $T_1, T_2 > 0$ such that

$$\frac{\mu_1}{2} T_1^2 < F(T_1) \quad \text{and} \quad \frac{\mu_2}{2} T_2^2 < G(T_2),$$

where $F(t) := \int_0^t f(s)\,\mathrm{d}s$ and $G(t) := \int_0^t g(s)\,\mathrm{d}s$.

We note that in the above conditions (A_1)–(A_3) neither any monotonicity condition nor any Ambrosetti–Rabinowitz growth condition is required. In the celebrated paper [6], this kind of conditions (A_1)–(A_3) appeared for the first time in the study of the nonlinear scalar field equation

$$-\Delta u + \mu u = f(u), \quad u \in H^1(\mathbb{R}^N). \tag{1.2}$$

Using minimizing arguments, Berestycki and Lions proved in [6] that there exists a positive radial least energy solution of (1.2) if f satisfies the conditions (A_1)–(A_3), and also, using Pohozaev identity they showed that these conditions are almost necessary to get an existence result for equation (1.2). In addition, Jeanjean and Tanaka [16] showed that the least energy solution of (1.2) is a mountain-pass solution.

Problem (1.1) is often referred to as nonlocal because of the appearance of the terms $(\int_{\mathbb{R}^3} |\nabla u|^2 \,\mathrm{d}x)\Delta u$ and $(\int_{\mathbb{R}^3} |\nabla v|^2 \,\mathrm{d}x)\Delta v$ which imply that (1.1) is no longer a pointwise identity. This phenomenon provokes some mathematical difficulties which make the study of such problems particularly interesting. Besides, the type of system (1.1) appears in the nonlinear vibrations of an elastic string, and such systems also model several physical and biological systems where u describes a process which depends on the average of itself, such as the population density (see for example [17,24]). Recently, the case of a single Kirchhoff-type equation, say

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