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Boundedness, almost periodicity and stability of certain Navier–Stokes flows in unbounded domains

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Abstract

We investigate the existence, uniqueness and stability of bounded and almost periodic mild solutions to several Navier–Stokes flow problems. In our strategy, we propose a general framework for studying the semi-linear evolution equations with certain smoothing properties of the linear part and with the local Lipschitz continuity of the nonlinear operator. Our method is based on interpolation functors combined with differential inequalities. Our abstract results are applied to Navier–Stokes–Oseen equations describing flows of incompressible viscous fluid passing a translating and rotating obstacle and to Navier–Stokes equations on aperture domains and/or in Besov spaces.

Keywords: Bounded solutions and stability; Almost periodicity; Navier–Stokes–Oseen equations; Navier–Stokes flows in Besov Spaces; Oseen operator; Stokes operator

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1. Introduction and preliminaries

We consider the following equations describing an incompressible viscous fluid flow passing a rotating and translating rigid body $\mathcal{R}$ with translational velocity $\xi = k\mathbf{e}_3$ and the (rotational) angular velocity w being both constant vectors:

$$\begin{cases} u_t + (u\cdot\nabla)u - \Delta u + kD_3u & \\ -((\omega\times x)\cdot\nabla)u + \omega\times u + \nabla p = \operatorname{div} F & \text{in } \Omega\times(0,\infty), \\ \operatorname{div} u = 0 & \text{in } \Omega\times(0,\infty), \\ u(x,t) = \omega\times x - k\mathbf{e}_3 & \text{on } \partial\Omega\times(0,\infty), \\ u(x,0) = u_0(x) := v_0(x) - \xi & \text{in } \Omega, \\ \lim_{|x|\to\infty} u(x,t) = 0 & \text{for all } t\in(0,\infty). \end{cases} \tag{1.1}$$

Here, the domain Ω is an exterior domain being the complement of $\mathcal{R}$ in $\mathbb{R}^3$. More detailed treatments of this problem will be presented in Subsection 4.1 of Section 4.

The existence and uniqueness of solutions to Eq. (1.1) have been investigated by many authors using various methods, namely, Shibata [33,34] with methods from semigroup theory and evolution equations, Galdi and Silvestre [12,13] with energy estimates and Galerkin's methods, and many others (see [14,26,28] and references therein). Meanwhile, we would like to emphasize that the difficulty that ones face when studying the existence of bounded (in time) solutions to Navier–Stokes equations on exterior domains is lying in the fact that, since the domain is unbounded in all directions, the Poincaré inequality in no longer valid and compact embeddings do not hold true. Furthermore, the semigroup generated by Stokes operator in such domains is not exponentially stable. Therefore, several authors have introduced new approaches to overcome these difficulties. We refer to Maremonti [28] and Maremonti–Padula [29] for the approach of using geometric properties of the domains such as the symmetry of Ω and the smallness of the complement $\mathbb{R}^d \setminus \Omega$. Meanwhile, Galdi and Sohr introduced in [14] some relevant function spaces featuring the decay of the solutions at spatial infinity to prove the existence of bounded and periodic solutions to Navier–Stokes equations on an exterior domain without restricted conditions on the domain. In the present paper, we investigate the existence of bounded (in time) solutions to the Eq. (1.1) and some other fluid flow problems and their stability and almost periodicity. To do that, we start with a general framework of semi-linear evolution equations and use the interpolation spaces combined with the smoothing properties of semigroups corresponding to linearized equations. Such a method has been introduced by Yamazaki [37]. He exploited interpolation properties of weak-L^d and Lorentz spaces and the Kato-iteration scheme [17,21] to prove the existence of bounded and time periodic mild solutions to Navier–Stokes equations on fixed exterior domains as well as their stability. This approach has then been extended by Nguyen [31] (combined with an ergodic method) to obtain existence and stability of time periodic mild solutions in interpolation spaces of Navier–Stokes equations around rotating obstacle. Such an approach has also been extended to prove the existence of periodic solutions to more general fluid flow problems in Geissert, Hieber and Nguyen [16].

We next would like to mention about the aperture domain which is a particularly interesting class of domains with noncompact boundaries. Heywood [18] have pointed out that the solutions of Navier–Stokes equations on an aperture domain may not be uniquely by usual boundary conditions. Therefore in order to get a unique solution one needed to impose the so-called "flux

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