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Approximate controllability for a one-dimensional wave equation with the fixed endpoint control

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Abstract

This paper is devoted to the study of the approximate controllability for a one-dimensional wave equation in domains with moving boundary. This equation models the motion of a string where an endpoint is fixed and the other one is moving. When the speed of the moving endpoint is less than the characteristic speed, the controllability of this equation is established. We present the following results: the existence and uniqueness of Nash equilibrium, the approximate controllability with respect to the leader control, and the optimality system for the leader control.

MSC: 35Q10; 35B37; 35B40

Keywords: Hierarchical control; Stackelberg strategy; Approximate controllability; Optimality system

1. Introduction

The development of science and technology has motivated many branches of control theory. Initially, in the classical control theory, we encountered problems where a system must reach a predetermined target by the action of a single control, for example, find a control of minimum norm such that the design specifications are met. To the extent that more realistic situations were considered, it was necessary to include several different (and even contradictory) control objectives, as well as develop theory that would handle the concepts of multi-criteria optimization,

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where optimal decisions need to be taken in the presence of trade-offs between these different objectives. There are many points of view to deal with multi-objective problems. Notions of economics and game theory were introduced in the works of J. Nash [1], V. Pareto [2] and H. von Stackelberg [3], where each has a particular philosophy to solve these problems.

According to the formulation introduced by H. von Stackelberg [3], we assume the presence of various local controls, called *followers* which have their own objectives, and a global control, called *leader*, with a different goal from the rest of the players (in the case, the followers). The general idea of this strategy is a game of hierarchical nature, where players compete against each other, so that the leader makes the first move and then followers react optimally to the action of the leader. Since many followers are present and each has a specific objective, it is intended that these are in Nash equilibrium.

This paper was inspired by ideas of J.-L. Lions [4], where we investigate a similar question of hierarchical control employing the Stackelberg strategy in the case of time dependent domains.

Up to date, in the context of partial differential equations (PDEs), there are several papers related to this topic. The papers by Lions [5,6], the author gives some results concerning Pareto and Stackelberg strategies, respectively. The paper by Díaz and Lions [7], the approximate controllability of a system is established following a Stackelberg–Nash strategy and the extension in Díaz [8], that provides a characterization of the solution by means of Fenchel–Rockafellar duality theory. In [9,10], Glowinski, Ramos and Periaux analyze the Nash equilibrium for constraints given by linear parabolic and Burger's equations from the mathematical and numerical viewpoints. The Stackelberg–Nash strategy for the Stokes systems has been studied by González, Lopes and Medar in [11]. In Limaco, Clark and Medeiros [12], the authors present the Stackelberg–Nash equilibrium in the context of linear heat equation in non-cylindrical domains. The paper by Araruna, Fernández-Cara and Santos [13], the authors developed the first hierarchical results within the exact controllability framework for a parabolic equation. Finally, we can mention the paper by Ramos and Roubicek [14], where the existence of a Nash equilibrium is proved for a nonlinear distributed parameter predator–prey system and a conceptual approximation algorithm is proposed and analyzed.

In this paper, we present the following results: the existence and uniqueness of Nash equilibrium, the approximate controllability with respect to the leader control, and the optimality system for the leader control.

1.1. Organization of the paper

The remainder of the paper is organized as follows. In Section 2, we present the problem. Section 3 is devoted to establish the existence and uniqueness of Nash equilibrium. In Section 4, we study the approximate controllability with respect to the leader control. Finally, in the Section 5 we present the optimality system for the leader control.

2. Problem formulation

As in [15], given $T > 0$, we consider the non-cylindrical domain defined by

$$\widehat{Q} = \left\{ (x,t) \in \mathbb{R}^2;\ 0 < x < \alpha_k(t),\ \ t \in (0,T) \right\},$$

where