ARTICLE IN PRESS

YJDEQ:8881

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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Distance of attractors of reaction-diffusion equations in thin domains ☆

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Received 22 April 2017

Abstract

In this work we consider a dissipative reaction–diffusion equation in a d-dimensional thin domain shrinking to a one dimensional segment and obtain good rates for the convergence of the attractors. To accomplish this, we use estimates on the convergence of inertial manifolds as developed previously in [6] and Shadowing theory.

Keywords: Thin domain; Attractors; Inertial manifolds; Shadowing

1. Introduction

In this work we study the rate of convergence of attractors for a reaction-diffusion equation in a thin domain when the thickness of the domain goes to zero. Our domain is a thin channel obtained by shrinking a fixed domain $Q \subset \mathbb{R}^d$, see Fig. 1, by a factor ε in $(d-1)$-directions. The thin channel Q_ε collapses to the one dimensional line segment $[0, 1]$ as ε goes to zero.

☆ This research has been partially supported by grants MTM2016-75465, MTM2012-31298, ICMAT Severo Ochoa project SEV-2015-0554 (MINECO), Spain and Grupo de Investigación CADEDIF, UCM.

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http://dx.doi.org/10.1016/j.jde.2017.06.023
0022-0396/

Please cite this article in press as: J.M. Arrieta, E. Santamaría, Distance of attractors of reaction-diffusion equations in thin domains, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.06.023

The reaction diffusion-equation in Q_ε is given by

$$\begin{cases} u_t - \Delta u + \mu u = f(u) & \text{in} \quad Q_\varepsilon, \\ \frac{\partial u}{\partial \nu_\varepsilon} = 0 & \text{in} \quad \partial Q_\varepsilon, \end{cases} \tag{1.1}$$

where $\mu > 0$ is a fixed number, ν_ε the unit outward normal to ∂Q_ε and $f : \mathbb{R} \to \mathbb{R}$ is a nonlinear term, with appropriate dissipativity conditions to guarantee the existence of an attractor $\mathcal{A}_\varepsilon \subset H^1(Q_\varepsilon)$.

As the parameter $\varepsilon \to 0$, the thin domain shrinks to the line segment $[0, 1]$ and the limiting reaction–diffusion equation is given by

$$\begin{cases} u_t - \frac{1}{g}(g u_x)_x + \mu u = f(u) & \text{in} \quad (0,1), \\ u_x(0) = u_x(1) = 0, \end{cases} \tag{1.2}$$

which also has an attractor $\mathcal{A}_0 \subset H^1(0, 1)$.

There are several works in the literature comparing the dynamics of both equations and showing the convergence of $\mathcal{A}_\varepsilon$ to $\mathcal{A}_0$ as $\varepsilon \to 0$, under certain hypotheses. One of the most relevant and pioneer work in this direction is [15], where the authors show that when $d = 2$ and every equilibrium of the limit problem (1.2) is hyperbolic, the attractors behave continuously and moreover, the flow in the attractors of both systems are topologically conjugate. In order to accomplish this task, the authors exploit the fact that the limit problem is one dimensional, which allows them to construct inertial manifolds for (1.1) and (1.2) which will be close in the C^1 topology. Restricting the flow to these inertial manifolds, and using that the limit problem is Morse–Smale (under the condition that all equilibria being hyperbolic, see [17]) they prove the C^0-conjugacy of the flows. Moreover the method of constructing the inertial manifolds for fixed $\varepsilon \in [0, \varepsilon_0]$ consists in using the method described in [20]. They consider the finite dimensional linear manifold given by the span of the eigenfunctions corresponding to the first m eigenvalues of the elliptic operator and let evolve this linear manifold with the nonlinear flow, which ω-limit set is a C^1 manifold and it is the inertial manifold, which, as a matter of fact it is a graph over the finite dimensional linear manifold. This method provides them with an estimate of the distance of the inertial manifolds of the order of ε^γ for some $\gamma < 1$. Later on, reducing the system to the inertial manifolds and using the general techniques to estimate the distance of attractors for gradient flows, see [14], Theorem 2.5, give them the estimate $\varepsilon^{\gamma'}$ with some $\gamma' < \gamma < 1$ which depends on the number of equilibria of the limit problem and other characteristics of the problem. We refer to [23] for a general reference on PDE's in thin domains. See also [22] for a different kind of thin domain. We refer to [12,8,19,27,13,26,10,28] for general references on infinite dimensional dynamical systems.

Our setting is more general than the one from [15], since we consider general d-dimensional thin domains (not just 2-dimensional). Moreover, our approach to this problem has some differences with respect to theirs. In our case, we will also construct inertial manifolds, but we will construct them following the Lyapunov–Perron method, as developed in [26]. This method, as it is shown in [6,7] provides us with good estimate of the C^0 distance of the inertial manifolds (which is of order $\varepsilon|\ln(\varepsilon)|$, see [6]) and with the $C^{1,\theta}$ convergence of this manifolds, see [7]. We refer to [26,9,11,25] for references on invariant and inertial manifolds.

Once the Inertial Manifolds are constructed and we have a good estimate of its distance we can project the systems to these inertial manifolds and obtain the reduced systems, which are finite dimensional. The limit reduced system will be a Morse–Smale gradient like system, see [12]. Then