ARTICLE IN PRESS YJDEQ:8887

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Existence, uniqueness, stability and bifurcation of periodic patterns for a seasonal single phytoplankton model with self-shading effect

Manjun Ma [a,*], Chunhua Ou [b,*]

[a] Department of mathematics, School of Science, Zhejiang Sci-Tech University, Hangzhou, Zhejiang, 310018, China

[b] Department of Mathematics and Statistics, Memorial University, St. John's, A1C 5S7, Canada

Received 23 February 2017; revised 22 June 2017

Abstract

We study the existence, uniqueness, global attractivity and bifurcation of time-periodic patterns for a seasonal phytoplankton model with self-shading effect. By the comparison principle, we obtain the globally asymptotical stability of the zero solution when the principal eigenvalue λ_1 is less than zero. When $\lambda_1 > 0$, by transforming the model into a new system, we successfully prove the conjecture in previous studies on the uniqueness and attractivity of the positive periodic solution. The positive periodic pattern bifurcating from the zero solution is a very interesting phenomenon. Here we apply the Crandall and Rabinowiz's theory to prove rigorously the existence of a bifurcation point. By way of asymptotic analysis, we derive an asymptotic formula for the positive periodic pattern. Based on the solution formula, we find the linear stability of this positive pattern. Finally, we provide a numerical scheme for the calculations of the principal eigenvalue and the simulations of the solution. The simulations corroborate our theoretical analysis.

MSC: 35K57; 35B20; 92D25

Keywords: Phytoplankton model; Periodic solution; Uniqueness; Global attractivity; Bifurcation; Stability

* Corresponding authors.
E-mail addresses: mjunm9@zstu.edu.cn (M. Ma), ou@mun.ca (C. Ou).

http://dx.doi.org/10.1016/j.jde.2017.06.029

Please cite this article in press as: M. Ma, C. Ou, Existence, uniqueness, stability and bifurcation of periodic patterns for a seasonal single phytoplankton model with self-shading effect, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.06.029

1. Introduction

Since the pioneer work of Riley, Stommel and Bumpus [19], mathematical modeling of phytoplankton bloom has attracted considerable attention of researchers, see [1,7,8,11,12,14, 18–20]. A simple convection–diffusion model was proposed by Shigesada and Okubo [20] which incorporates the sinking and self-shading effect of the phytoplankton. Let $u(x,t)$ denote the population density of plankton at depth x and time t. The original model of Shigesada and Okubo reads

$$\begin{cases} u_t = Du_{xx} - \alpha u_x + [g(I(x,t)) - d]u, & x \in (0,\infty),\ t > 0, \\ Du_x - \alpha u = 0, & x = 0,\ t > 0, \\ u(x,0) = u_0(x) \geq 0, & x \in [0,\infty), \end{cases} \tag{1.1}$$

where D is the diffusivity and α is the sinking velocity. $g(I(x,t))$ is the growth rate of the phytoplankton as a function of light intensity $I(x,t)$. Typical functions for g are given by

$$g(I) = \frac{aI}{1+bI}, \quad \text{or} \quad g(I) = b\frac{1-e^{-cI}}{c},$$

for some constants a, b and c, see also [15]. Due to the absorption of water and the self-shading effect of phytoplankton, the light intensity is modeled by

$$I(x,t) = I_0 e^{-k_0 x - k_1 \int_0^x u(s,t)ds},$$

where I_0 is the light intensity at the surface, k_0 and k_1 are two constants. The water depth is assumed sufficiently large so that another boundary condition at the bottom is added as

$$\lim_{x\to\infty} u(x,t) = 0. \tag{1.2}$$

The system (1.1)–(1.2) is a non-local convection diffusion model. If the phytoplankton is assumed to be sufficiently transparent (i.e., $k_1 = 0$), (1.1) reduces to a linear model which was investigated in [8]. Fennel in [8] found that the species attains its peak density at the vertical location where the growth rate and the death rate are balanced. When water is assumed to be sufficiently transparent (i.e., $k_0 = 0$) so that all of the light is absorbed by the phytoplankton itself, this is called a completely self-shading model. Steady-state solutions of the model were investigated by the phase plane method in [20]. When neither k_0 nor k_1 is equal to zero, the global stability of the stationary solutions were studied by way of comparison and energy method in [13].

It is interesting to study the model when the water depth is finite. In [15], Kolokolnikov, Ou and Yuan added a non-flux boundary condition at the bottom with depth L as

$$Du_x - \alpha u = 0,\ \ x = L,\ \ t > 0. \tag{1.3}$$

When $k_0 = 0$, the phytoplankton depth profiles and their transitions near the critical sinking velocity were studied. Depending on the sinking rate, light intensity and water depth, the plankton can concentrate either near the surface, or at the bottom of the water column, or both, resulting