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Propagation phenomena in monostable integro-differential equations: Acceleration or not?

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Abstract

We consider the homogeneous integro-differential equation $\partial_t u = J * u - u + f(u)$ with a monostable nonlinearity f. Our interest is twofold: we investigate the existence/nonexistence of travelling waves, and the propagation properties of the Cauchy problem.

When the dispersion kernel J is exponentially bounded, travelling waves are known to exist and solutions of the Cauchy problem typically propagate at a constant speed [22,26,7,11,10,27]. On the other hand, when the dispersion kernel J has heavy tails and the nonlinearity f is nondegenerate, i.e. $f'(0) > 0$, travelling waves do not exist and solutions of the Cauchy problem propagate by accelerating [20,27,14]. For a general monostable nonlinearity, a dichotomy between these two types of propagation behaviour is still not known.

The originality of our work is to provide such dichotomy by studying the interplay between the tails of the dispersion kernel and the Allee effect induced by the degeneracy of f, i.e. $f'(0) = 0$. First, for algebraic decaying kernels, we prove the exact separation between existence and nonexistence of travelling waves. This in turn provides the exact separation between nonacceleration and acceleration in the Cauchy problem. In the latter case, we provide a first estimate of the position of the level sets of the solution.

Contents

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1. Introduction

In this work, we are interested in the propagation phenomena for solutions $u(t,x)$ of homogeneous integro-differential equations of the form

$$\partial_t u = J * u - u + f(u), \quad t > 0,\ x \in \mathbb{R}. \tag{1}$$

In population dynamics models, u stands for a population density, the nonlinearity f encodes the demographic assumptions and J is a nonnegative dispersal kernel of total mass 1, allowing to take into account long distance dispersal events. Here, we consider nonlinearities f of the monostable type, namely $f(0) = f(1) = 0$ and $f > 0$ on $(0,1)$. Precise assumptions on J (heavy tails) and f (degeneracy at 0) will be given later on.

When f is nondegenerate at 0, that is $f'(0) > 0$, it is known that the equation (1) exhibits some propagation phenomena: starting with some nonnegative nontrivial compactly supported initial data, the corresponding solution $u(t,x)$ converges to 1, its stable steady state, at large time and locally uniformly in space. This is referred as the *hair trigger effect* [4]. Moreover, in many cases, the convergence to 1 can be precisely characterised. For example, when f is a KPP nonlinearity — meaning $f(s) \le f'(0)s$ for all $s \in (0,1)$ — and J is exponentially bounded, that is

$$\exists \lambda > 0, \quad \int_{\mathbb{R}} J(z) e^{\lambda |z|}\, dz < +\infty, \tag{2}$$

equation (1) admits travelling waves whose minimal speed c^* completely characterises the convergence $u(t,x) \to 1$, see [22,26,7,11,10].

For nondegenerate monostable nonlinearities f, when the condition (2) is relaxed, allowing dispersion kernels with *heavy tails*, a new propagation phenomena appears: *acceleration*. This phenomenon for equation (1) was first heuristically obtained by Medlock and Kot [20] and mathematically described in [27,14]: Yagisita [27] proves the nonexistence of travelling waves, and Garnier [14] studies the acceleration in the Cauchy problem.

Remark 1.1. Acceleration phenomena for positive solutions of a Cauchy problem also appear in other contexts ranging from standard reaction diffusion equations [16,2], to homogeneous equations involving fractional operators [6]. Let us also mention that acceleration phenomenon also appears in some porous media equations [18,23].

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