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Boundedness and global solvability to a chemotaxis model with nonlinear diffusion

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Abstract

In this paper, we study a chemotaxis model with nonlinear diffusion Δu^m $(m > 1)$. We consider this problem in a bounded domain $\Omega \subset \mathbb{R}^3$ with zero-flux boundary condition, and it is shown that for any large initial datum, for any $m > 1$, the problem admits a global weak solution, which is uniformly bounded.

MSC: 92C17; 35K40; 35K55

Keywords: Chemotaxis system; Nonlinear diffusion; Logistic source; Boundedness

1. Introduction

In this paper, we consider the following chemotaxis model

$$\begin{cases} u_t = \Delta u^m - \nabla \cdot (u \cdot \nabla v) + \mu u(1-u), \text{ in } Q, \\ v_t - \Delta v = -vu, \text{ in } Q, \\ (\nabla u^m - u \cdot \nabla v) \cdot \mathbf{n}\big|_{\partial\Omega} = \dfrac{\partial v}{\partial \mathbf{n}}\Big|_{\partial\Omega} = 0, \\ u(x,0) = u_0(x), v(x,0) = v_0(x), \quad x \in \Omega, \end{cases} \tag{1.1}$$

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where $m > 1$, $Q = \Omega \times \mathbb{R}^+$, $\Omega \subset \mathbb{R}^3$ is a bounded domain, and the boundary $\partial\Omega$ is appropriately smooth, u, v represent the bacterial density, the chemoattractant concentration respectively, $J = u \cdot \nabla v$ is the chemotactic flux, $\mu u(1-u)$ ($\mu > 0$) is the proliferation or death of bacteria according to a generalized logistic law, $-vu$ is the consumption of chemoattractant.

Chemotaxis model was first introduced by Keller and Segel [5] in 1970, since then, a lots of modified chemotaxis models have been widely investigated by many researchers. For the Keller–Segel model of this form

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \cdot \nabla v), \\ \tau v_t - \Delta v + v = u, \end{cases}$$

in which, the chemical not only be consumed, but also be produced by cells. It is well known that the formation of cell aggregates will result in the finite-time blow-up of cell density in spatial dimensions $N \geq 2$, see for example [1,2,11,18]. When a logistic growth term $\mu u(1-u)$ characterizing the death of proliferation of cells is introduced into this model. It is found that the logistic growth term will weaken the aggregation behavior, and prevent blow up, for instance, in two-dimensional space, the solutions will always exist globally [16]; and in three-dimensional space, the solutions will exist globally for large μ [16,21]. For the nonlinear diffusion case, we refer to [8,22].

While if only the consumption of the cells is considered, that is for the Keller–Segel model of this form

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \cdot \nabla v) + \mu u(1-u), \\ v_t - \Delta v = -vu. \end{cases} \tag{1.2}$$

When $\mu = 0$, it is shown that for arbitrarily large initial datum, this problem admits a unique global classical solution in two-dimensional space, admits a global weak solution in three-dimensional space, and more interesting fact is that, the weak solution will become smooth after time T [13]; When $\mu > 0$, in three dimensional case, Zheng, Mu [23] showed that the system (1.2) admits a unique global classical solution if the initial datum of v is small; Lankeit and Wang [7] obtained the global classical solutions for large μ, but without the smallness assumption on the initial data, and for any $\mu > 0$, they also established the existence of global weak solutions. Lankeit also considered a coupled system of (1.2) and the incompressible Navier–Stokes equations [6], in which, the global existence of weak solutions are proved, however, after some time the weak solutions will become smooth and finally converge to a semi-trivial steady state.

However, as indicated by Szymanska, Chaplain et al. [10], the equation modeling the migration of cells should rather be regarded as nonlinear diffusion, in which, the cell mobility is described by a nonlinear function of the cells density, for example, the porous medium diffusion. So, in recent years, many researchers are led to study the model with nonlinear diffusion. For the following system ($m > 1$),

$$\begin{cases} u_t = \Delta u^m - \nabla \cdot (u \cdot \nabla v), \\ v_t - \Delta v = -vu, \end{cases}$$

in two dimensional space, the global solvability of weak solution is established for any $m > 1$ [14]; in three dimensional space, a global bounded weak solution is obtained for $m > \frac{7}{6}$ [19],

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