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Local-in-space blow-up criteria for a class of nonlinear dispersive wave equations

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Abstract

This paper is concerned with blow-up phenomena for the nonlinear dispersive wave equation on the real line, $u_t - u_{xxt} + [f(u)]_x - [f(u)]_{xxx} + \left[g(u) + \frac{f''(u)}{2}u_x^2\right]_x = 0$ that includes the Camassa–Holm equation as well as the hyperelastic-rod wave equation ($f(u) = ku^2/2$ and $g(u) = (3-k)u^2/2$) as special cases. We establish some a local-in-space blow-up criterion (i.e., a criterion involving only the properties of the data u_0 in a neighborhood of a single point) simplifying and precising earlier blow-up criteria for this equation.

MSC: 35B44; 35L05; 37K10

Keywords: Rod equation; Blow-up; Shallow water

1. Introduction

In this paper, we consider the following Cauchy problem for the generalized hyperelastic rod wave equation

$$u_t - u_{xxt} + [f(u)]_x - [f(u)]_{xxx} + \left[g(u) + \frac{f''(u)}{2}u_x^2\right]_x = 0 \tag{1}$$

$$u(x,0) = u_0(x). \tag{2}$$

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This equation includes the Camassa–Holm equation [8], the hyperelastic-rod wave equation [23], and its generalization [30] (see also [10,26], for more specific choices of the functions f and g).

When $f(u) = u^2/2$ and $g(u) = ku + u^2$ (1) becomes the Camassa–Holm equation

$$u_t - u_{xxt} + ku_x + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \quad t > 0, x \in \mathbb{R}$$

where k is a dispersive coefficient related to the critical shallow water speed. The details concerning the hydrodynamical relevance of Camassa–Holm equation were mathematically rigorously described by Constantin and Lannes in [12], where, in addition, authors investigate in what sense model under consideration gives us insight into the wave breaking phenomenon. Alternative derivations of Camassa–Holm equation as a equation for geodesic flow on the diffeomorphism group of the circle were presented by Constantin and Kolev [11] and Ionescu-Kruse [31]. The equation has bi-Hamiltonian structure [25] and is completely integrable [1,2,9,13,17]. Note that local well-posedness for the initial datum $u_0(x) \in H^s$ with $s > 3/2$ was proved by several authors (see, for example, [19,35,37]). For the initial data with lower regularity, we refer to paper [6] and [38].

When $k = 0$, Camassa–Holm equation possesses a solitary waves with discontinuous first derivatives ([8]), which named "peakon" (travelling wave solutions with a corner at their peak). It is worth pointing out that these solutions resemble the Stokes waves of greatest height which arise as extreme travelling wave solutions to the governing equations for water waves in irrotational flow (see the discussion in the papers [14–16]). Interestingly, while in periodic waves within a period each particle experiences a backward–forward motion with a slight forward drift (see [15]), it was shown (see [16]) that in a solitary water wave there is no backward motion: all particles move in the direction of wave propagation at a positive speed, the direction being upwards/downwards if the particle precedes or does not precede the wave crest. This shows that in the long-wave limit the shapes of the periodic waves approach the profile of a solitary wave but the pattern of the particle motion within the fluid is not preserved in this limiting process. Also note that, the peakons are orbitally stable ([21,34]), which means that the shape of the peakons is stable so that these wave patterns are physically recognizable.

Wave breaking for a large class of initial data has been established in [19,20,35,43,44] and the recent paper [32], where, in particular, new and direct proof for the result from [36] on the necessary and sufficient condition for wave breaking was presented. The fact that after wave breaking the solution of the Camassa–Holm equation can be continued uniquely as either global conservative [6] or global dissipative solutions has been noticed by Bressan and Constantin [7]. It is also worth mentioning that from the point of view of theory of water waves the fact that solutions that originate from smooth localized initial data can develop singularities only in the form of breaking waves, as proved in the paper [18], is especially interesting. The infinite propagation speed for the Camassa–Holm equation for $k = 0$ was investigated in [22,28,29] (see also [40] for $k \neq 0$). Also, the wide range of problems for CH equation with non-zero dispersion coefficient was considered in [35,45,40,39]. In particular, certain conditions on the initial datum to guarantee that the corresponding solution exists globally or blows up in finite time were established.

For $f(u) = ku^2/2$ and $g(u) = (3 - k)u^2/2$, (1) becomes the hyperelastic rod wave equation

$$u_t - u_{xxt} + 3uu_x - \gamma\,(2u_x u_{xx} + uu_{xxx}) = 0 \tag{3}$$

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