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# Blow-up phenomena and persistence properties for an integrable two-component peakon system

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**Abstract**

In this paper, we are concerned with an integrable two-component peakon system, which was proposed by Xia, Qiao and Zhou. We present a precise blow-up scenario and a new blow-up result for strong solutions to the system. Moreover, we prove that the strong solutions of the system maintain corresponding properties at infinity within its lifespan provided the initial data decay exponentially and algebraically, respectively.


*Keywords:* Two-component peakon system; Blow-up; Persistence properties; Exponential decay; Algebraical decay

## 1. Introduction

In this paper, we consider an integrable two-component peakon system as follows:

$$\begin{cases} m_t = \frac{1}{2}[m(uv_x - u_x v)]_x - \frac{1}{2}m(uv - u_x v_x), \\ n_t = \frac{1}{2}[n(uv_x - u_x v)]_x + \frac{1}{2}n(uv - u_x v_x), \end{cases} \tag{1.1}$$

where $m = u - u_{xx}$ and $n = v - v_{xx}$. The system (1.1) was proposed by Xia, Qiao and Zhou in [5], in which they show that the system (1.1) can be rewritten as the following bi-Hamiltonian form:

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$$(m_t, n_t)^T = J\left(\frac{\delta H_2}{\delta m}, \frac{\delta H_2}{\delta n}\right)^T = K\left(\frac{\delta H_1}{\delta m}, \frac{\delta H_1}{\delta n}\right)^T,$$

where

$$J = \begin{pmatrix} 0 & \partial^2 - 1 \\ 1 - \partial^2 & 0 \end{pmatrix}, \quad K = \begin{pmatrix} \partial m \partial^{-1} m \partial - m \partial^{-1} m & \partial m \partial^{-1} n \partial + m \partial^{-1} n \\ \partial n \partial^{-1} m \partial + n \partial^{-1} m & \partial n \partial^{-1} n \partial - n \partial^{-1} n \end{pmatrix},$$

$$H_1 = \frac{1}{2}\int_{-\infty}^{+\infty} (uv_x + u_x v_{xx}) dx, \quad H_2 = \frac{1}{4}\int_{-\infty}^{+\infty} (uv + u_x^2 v - 2uu_x v_x) n dx,$$

and admit the following one-peakon solution:

$$u(t,x) = c_2 e^{-\frac{1}{3}c_1 t} e^{-|x-c_3|}, \quad v(t,x) = \frac{c_1}{c_2} e^{\frac{1}{3}c_1 t} e^{-|x-c_3|},$$

where $c_1, c_2$ and $c_3$ are constants. Moreover, N-peakon solutions and peakon interactions were discussed (for details, see [5]). Recently, the local well-posedness of the system (1.1) has been studied in the Besov spaces by using Littlewood–Paley decomposition and transport equation theory [1].

For $v = 2$, the system (1.1) is reduced to the Camassa–Holm (CH) equation:

$$m_t + 2u_x m + u m_x = 0, \quad m = u - u_{xx}, \tag{1.2}$$

which was first derived formally by Fokas and Fuchssteiner in [6], and later derived as a model for unidirectional propagation of shallow water over a flat bottom by Camassa and Holm in [7]. Moreover, the CH equation could be also derived as a model for the propagation of axially symmetric waves in hyperelastic rods [8]. The CH equation has a bi-Hamilton structure [6] and is completely integrable [7,9]. In particular, it possesses an infinity of conservation laws and is solvable by its corresponding inverse scattering transform. After the birth of the CH equation, many works have been carried out to it. For example, the CH equation has traveling wave solutions of the form $ce^{-|x-ct|}$, called peakons, which describes an essential feature of the traveling waves of largest amplitude [10–12]. It is shown in [13] that the blow-up occurs in the form of breaking waves, namely, the solution remains bounded but its slope becomes unbounded infinite time. Moreover, the CH equation has global conservative solutions [14] and dissipative solutions [15]. The orbital stability of solitary waves and the stability of peakons for the CH equation are considered by Constantin and Strauss [16,17]. For the related generalized Camassa–Holm equation, we refer to [18–30] and the references therein.

Noticing that the system (1.1) can be regarded as generalized version of the two-component peakon system:

$$\begin{cases} m_t = (mH)_x + mH - \frac{1}{2}m(u - u_x)(v + v_x), \\ n_t = (nH)_x - nH + \frac{1}{2}n(u - u_x)(v + v_x), \end{cases} \tag{1.3}$$

where $m = u - u_{xx}$, $n = v - v_{xx}$ and $H$ is an arbitrary function of $u, v$ and their derivatives. If one chooses suitable function $H$, the system (1.3) can be reduced to some of integrable peakon systems which possess Lax pair and infinitely many conservation laws.