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Spectrum and bifurcation for semilinear elliptic problems in $\mathbb{R}^N$ ☆

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Abstract

This paper is concerned with the following semilinear elliptic problem

$$\begin{cases} -\Delta u = \lambda m(x) f(u) & \text{in } \mathbb{R}^N, \\ u \to 0 & \text{as } |x| \to +\infty, \end{cases}$$

where λ is a real parameter and m is a weight function which may be sign-changing. For the linear case, i.e., $f(u) = u$, we investigate the spectral structure. For the semilinear case, we study the existence and asymptotic behavior of one-sign and nodal solutions by bifurcation method.

MSC: 35B20; 35B32; 35B40; 35J60; 35P05

Keywords: Bifurcation; Spectrum; Nodal solutions

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1. Introduction

Consider the following eigenvalue problem

$$\begin{cases} -\Delta u = \lambda m(x) u & \text{in } \mathbb{R}^N, \\ u \to 0 & \text{as } |x| \to +\infty, \end{cases} \tag{1.1}$$

where $N \geq 3$, λ is a real parameter and $m \in C^{\alpha}_{\mathrm{loc}}\left(\mathbb{R}^N, \mathbb{R}\right)$ for some $\alpha \in (0,1)$ is a weighted function which can be sign-changing.

If there exist two continuous radially symmetric functions p and P such that

$$0 < p(|x|) \leq m(x) \leq P(|x|) \text{ for all } x \in \mathbb{R}^N$$

and

$$\int_{\mathbb{R}^N} |x|^{2-N} P(|x|)\, dx < +\infty, \tag{1.2}$$

then Edelson and Rumbos [5] have shown that problem (1.1) has a positive, simple, principal eigenvalue λ_1. Furthermore, if P satisfies the following more strong condition (with $r = |x|$)

$$\int_0^{+\infty} r^{N-1} P(r)\, dr < +\infty, \tag{1.3}$$

the positive principle eigenfunction ϕ satisfies the asymptotic decay law

$$\lim_{|x| \to +\infty} |x|^{N-2} \phi(x) = c$$

for some positive constant c. Besides the above important results, Edelson and Rumbos [5, 12], Edelson and Furi [6] also obtained some interesting results involving existence of positive minimal solution by the Schauder–Tychonoff fixed point theorem or the Rabinowitz global bifurcation theorem [11] for the following semilinear elliptic problem

$$\begin{cases} -\Delta u = \lambda m(x) f(u) & \text{in } \mathbb{R}^N, \\ u \to 0 & \text{as } |x| \to +\infty, \end{cases} \tag{1.4}$$

where $f \in C^{\alpha}(\mathbb{R}, \mathbb{R})$.

If $m(x)$ is radially symmetric and satisfies $m(r) \geq 0$ (with $r = |x|$) on $[0, +\infty)$, $m(r) \not\equiv 0$ on $[T, +\infty)$ for every $T \geq 0$ and

$$\int_0^{+\infty} r^{N-1} m(r)\, dr < +\infty,$$

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