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Geometric methods of global attraction in systems of delay differential equations

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Abstract

In this paper we deduce criteria of global attraction in systems of delay differential equations. Our methodology is new and consists in "dominating" the nonlinear terms of the system by a scalar function and then studying some dynamical properties of that function. One of the crucial benefits of our approach is that we obtain delay-dependent results of global attraction that cover the best delay-independent conditions. We apply our results in a gene regulatory model and the classical Nicholson's blowfly equation with patch structure.

Keywords: Dominance; Delay dependent criteria; Nicholson's blowfly equation; Global attraction

1. Introduction

Systems of delay differential equations, or more generally functional differential equations, have been extensively used as mathematical models for understanding medical treatments, the chemical patterns formation, ecological dynamics, cellular processes and control theory; see [1,2,10–12,16,21] and the references therein. Time delays are generally an inherent trait in a broad range of situations. For instance, they arise in population dynamics when the maturation period

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of species is taken into account [8,21]. Delays are also inevitable in neuroscience since the information from a neuron always reaches the other after a certain time by the finite propagation speed [3]. Delayed feedbacks can not be omitted in tumor systems and epidemiology [1,16]. Such delays offer the possibility of introducing the time required for the molecular production, proliferation and differentiation of cells; and incubation periods in epidemic scenarios.

In this paper, we study

$$x_i'(t) = -d_i x_i(t) + F_i(x_1(t-\tau_{i1}), ..., x_n(t-\tau_{in})) \quad 1 \leq i \leq n \tag{1.1}$$

where $d_i > 0$, $\tau_{ij} \geq 0$, and $F_i : \mathbb{R}^n \longrightarrow \mathbb{R}$ is a smooth map. The term $-d_i x_i(t)$ is a decay term in the i-th variable in the absence of new recruitment/activation/production; and the non-linear component F_i represents a delayed feedback. Many models, e.g. Hopfield's model, epidemical systems or age-structured population models, fall into system (1.1). A distinguished example is the classical Nicholson blowfly equation

$$x'(t) = -dx(t) + f(x(t-\tau)) \tag{1.2}$$

where $d > 0$ and $f(x) = rxe^{-x}$. This model was proposed by Gurney et al. [14,25] to explain the oscillatory behavior presented in the population of the sheep blowfly *Lucilia Cuprina*. In equation (1.2), $x(t)$ denotes the population of sexually adults at time t, d is the per-capita mortality rate and the birth function is expressed by f. The time delay τ is the time taken from eggs to sexually mature adults. The population oscillations in (1.2) are a direct consequence of the delay because the non-trivial equilibrium is always a global attractor when $\tau = 0$. In this particular case and in many others, increasing the magnitude of the time delay can destabilize equilibria and oscillations appear. In other systems, however, there can be several switches between stability and instability [17,21,28] or the dynamical behavior remains unaltered when the delay length is increased [2,20]. Certainly, the role of delay in system (1.1) depends on the interplay with the non-linear terms.

The aim of the paper is to provide criteria of global attraction of non-trivial equilibria in (1.1). In practical applications, the existence of a global attractor is very desirable because the long term behavior is easy to control and predict. The problem of global attractivity has been extensively studied and have motivated the development of a broad variety of analytical and geometrical techniques such as Lyapunov functions or the theory of monotone semiflows generated by functional differential equations [24,27]. But little work has been done on sharp delay-dependent criteria of global attraction. Here we initiate a systematic mechanism to approach this problem. Note that by the nature of (1.1), even determining the explicit expression of non-trivial equilibria is not an easy task. Our methodology is new and consists in "dominating" the non-linear terms of (1.1) by a scalar function. After that we study some dynamical properties of the discrete equation given by such a function. An important advantage is that our results apply to a wide variety of situations even when the usual linearization technique becomes unfeasible.

Our paper is organized as follows. The definition of dominance, the key aspect of our approach, is introduced in Section 2. Under such an assumption, we provide some delay-independent criteria of global attraction in system (1.1). We will see in the applications that our results generally give the best delay-independent condition of global attraction. In Section 3, we modify our method in order to deduce delay-dependent results of global attraction that cover the best delay-independent conditions. Since our approach involves some dynamical properties of discrete equations, Section 4 reviews some useful results of global attraction in this framework.