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Multiplicity results for magnetic fractional problems

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Abstract

The paper deals with the existence of multiple solutions for a boundary value problem driven by the magnetic fractional Laplacian $(-\Delta)^s_A$, that is

$$(-\Delta)^s_A u = \lambda f(|u|)u \text{ in } \Omega, \qquad u = 0 \text{ in } \mathbb{R}^n \setminus \Omega,$$

where λ is a real parameter, f is a continuous function and Ω is an open bounded subset of $\mathbb{R}^n$ with Lipschitz boundary. We prove that the problem admits at least two nontrivial weak solutions under two different sets of conditions on the nonlinear term f which are dual in a suitable sense.

1. Introduction

In the last years there has been an increasing interest in the study of equations driven by non-local operators. This is motivated by the fact that non-local operators appear naturally in many important problems in pure and applied mathematics. The prototype of non-local operator is the fractional Laplacian $(-\Delta)^s$ defined, up to normalization factors, for any $u \in C_0^\infty(\mathbb{R}^n)$ and $s \in (0,1)$ as

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$$(-\Delta)^s u(x) = \lim_{\varepsilon\to 0^+} \int\limits_{\mathbb{R}^n\setminus B(x,\varepsilon)} \frac{u(x)-u(y)}{|x-y|^{n+2s}} dy, \qquad x\in\mathbb{R}^n, \tag{1.1}$$

where $B(x,\varepsilon)$ denotes the ball of center x and radius ε. We refer to [7,16] and the references therein for further details on the fractional Laplacian.

In the present paper, we will focus on the so-called magnetic fractional Laplacian. This non-local operator has been recently introduced in [6,8] and can be considered as a fractional counterpart of the magnetic Laplacian $(\nabla - \mathrm{i}A)^2$, with $A:\mathbb{R}^n\to\mathbb{R}^n$ being a L^∞_{loc}-vector field, see [9]. We refer the interested reader to [6] for further details about the physical relevance of the magnetic fractional Laplacian. In [6], it has been proved that $(-\Delta)^s_A$ has the following representation when acting on smooth complex-valued functions $u\in C_0^\infty(\mathbb{R}^n,\mathbb{C})$

$$(-\Delta)^s_A u(x) = 2\lim_{\varepsilon\to 0^+} \int\limits_{\mathbb{R}^n\setminus B(x,\varepsilon)} \frac{u(x)-e^{\mathrm{i}(x-y)\cdot A\left(\frac{x+y}{2}\right)}u(y)}{|x-y|^{n+2s}} dy, \qquad x\in\mathbb{R}^n,$$

therefore, the operator is consistent with (1.1) if $A=0$. As for the classical fractional Laplacian, one can define the fractional counterpart of the magnetic Sobolev spaces, see Section 2 below for the definition. In [18,19,22], it has been studied the stability of these fractional Sobolev norms when either $s\nearrow 1$ or $s\searrow 0$, proving a magnetic counterpart of the Bourgain–Brezis–Mironescu formula (when $s\nearrow 1$, see [3]) and the Maz'ya–Shaposhnikova formula (when $s\searrow 0$, see [11,12]). Finally, we refer to [2,13,23] for multiplicity results for different equations on $\mathbb{R}^n$ and driven by the magnetic fractional Laplacian.

Inspired by the above-mentioned works, in this paper we study the existence of multiple weak solutions of the following boundary value problem

$$\begin{cases} (-\Delta)^s_A u = \lambda f(|u|)u, & \text{in } \Omega,\\ \qquad\qquad u = 0, & \text{in } \mathbb{R}^n\setminus\Omega, \end{cases} \tag{1.2}$$

where $\lambda\in\mathbb{R}$ and $\Omega\subset\mathbb{R}^n$ is an open and bounded set with Lipschitz boundary $\partial\Omega$.

Concerning the nonlinearity f, we will consider two different situations which can be considered *dual* in a sense that we will specify later on. As a first scenario, we will deal with $f:[0,\infty)\to\mathbb{R}$ being a *continuous* function satisfying the following conditions:

(f_1) $f(t)=o(1)$ *as* $t\to 0$;
(f_2) $f(t)=o(1)$ *as* $t\to\infty$;
(f_3) $\sup\limits_{t\in[0,\infty)} F(t)>0$,

where

$$F(t) := \int_0^t f(\tau)\,\tau\,d\tau, \quad \text{for any real } t>0. \tag{1.3}$$