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# Asymptotic homogenization in a three-dimensional nutrient taxis system involving food-supported proliferation

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**Abstract**

The taxis system

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + uv - \rho u, \\ v_t = D\Delta v - \xi uv + \mu v(1 - \alpha v), \end{cases} \tag{$\star$}$$

is considered in bounded convex domains $\Omega \subset \mathbb{R}^n$, $n \geq 1$, where $D$, $\chi$ and $\xi$ are positive but $\rho$, $\mu$ and $\alpha$ are merely assumed to be nonnegative. This particularly includes the special case $\rho = \mu = 0$ of a nutrient taxis system with food-dependent cell proliferation, but beyond that the system ($\star$) for general parameter choices is also used as a model for Lotka–Volterra-type interaction involving prey taxis.

It is firstly shown that if $n \leq 5$, then for all suitably regular initial data an associated no-flux initial-boundary value problem admits a global weak solution. To the best of our knowledge, this inter alia provides the first result on global existence in a system of the form ($\star$) in a spatially three-dimensional setting when arbitrarily large initial data and parameters are involved.

Secondly, under the additional hypotheses that $n \leq 3$, $\rho = 0$ and $\mu < \frac{16D\alpha}{\chi^2}$ it is seen that each of these solutions becomes eventually smooth and stabilizes toward a spatially homogeneous equilibrium in the sense that

$$u(\cdot, t) \to u_\infty \quad \text{in } L^\infty(\Omega) \qquad \text{and} \qquad v(\cdot, t) \to 0 \quad \text{in } L^\infty(\Omega) \tag{0.1}$$

as $t \to \infty$, where $u_\infty$ is a constant satisfying $u_\infty \geq \frac{1}{|\Omega|} \int_\Omega u_0$.

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This on the one hand shows that in comparison to the well-understood simple nutrient taxis system obtained on letting $\rho = \mu = 0$ and removing the proliferation term $+uv$ in $(\star)$, the introduction of the latter does not substantially affect the tendency of the system to support relaxation into spatial homogeneity. Apart from that, (0.1) complements previous findings on convergence to equilibria in prey-taxis systems of type $(\star)$, which seem to exclusively concentrate on cases in which unlike in (0.1) the constant limit functions are a priori known, and in which thereby the corresponding asymptotic analysis is apparently simplified to a considerable extent.


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## 1. Introduction

Orienting movement toward nutrients appears to be among the most vital abilities of individuals at virtually all levels of complexity in biology, and possible effects of such nutrient taxis mechanisms have been the objective of various experimental studies. Indeed, impressively colorful collective behavior could be observed even in quite simple setups involving populations of very primitive bacteria such as *Bacillus subtilis*, the motion of which is essentially known to be exclusively determined by random diffusion and cross-diffusive migration toward oxygen which they consume: The potential of such constellations to form complex spatial patterns has been known for a long time (see e.g. [4,9,8]), and more recent studies have revealed emergence of plume-like aggregates and large-scale convection patterns of such populations when suspended in sessile drops of water ([27]).

Up to now, however, it seems that mathematical models have been able to capture only part of this large dynamical variety. For instance, in the apparently simplest chemotaxis system that accounts for consumption of the chemoattractant, as given by

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v), \\ v_t = \Delta v - uv, \end{cases} \tag{1.2}$$

the solution behavior is known to be essentially dominated by diffusion, and hence to be rather simple, at least on large time scales: Namely, when posed along with no-flux boundary conditions and suitable assumptions on the regularity of the initial distributions in bounded convex domains $\Omega \subset \mathbb{R}^n$, in the case $n = 2$ (1.2) admits global classical solutions which satisfy

$$u(\cdot,t) \to \frac{1}{|\Omega|} \int_\Omega u_0 \quad \text{and} \quad v(\cdot,t) \to 0 \qquad \text{in } L^\infty(\Omega) \text{ as } t \to \infty; \tag{1.3}$$

in the three-dimensional counterpart, at least global weak solutions exist which eventually become smooth and stabilize according to (1.3) ([25,35]). Even an additional inclusion of buoyancy-driven interaction with a surrounding fluid does not essentially affect these asymptotic properties, so that within the context of (1.2) nontrivial dynamical aspects, if observable at all, seem excluded at least on large time scales.