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Rough flows and homogenization in stochastic turbulence

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Abstract

We provide in this work a tool-kit for the study of homogenisation of random ordinary differential equations, under the form of a friendly-user black box based on the technology of rough flows. We illustrate the use of this setting on the example of stochastic turbulence.

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Contents

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1. Introduction

The history of averaging and homogenization problems for dynamical systems is fairly long and has its roots in classical perturbative problems in mechanics, in the 19th century. It has evolved in an impressive body of methods and tools used to analyse a whole range of multi-scale systems, such as, possibly random, transport equations with multiple time-scales [1,2], or heat propagation in random media [3,4]. The latest developments of Otto, Gloria & co [5] and Armstrong & co [6–8] on homogenization for the solutions of elliptic equations use and develop deep results in partial differential equations. The present work deals with the transport side of the story, in the line of the classical works of Kesten and Papanicolaou on homogenization for random stochastic differential equations [9–11], and put them in the flow of ideas and tools that have emerged in the early 2000's with rough paths theory. Note that the idea of using rough path theory to deal with homogenization results has already been use is few works, both for PDEs and differential equations. (One can think of the early work of Marty [12] in the case of oscillatory periodic coefficients for stochastic differential equations, or the work [13] of Friz, Gassiat and T. Lyons on physical Brownian motion in a magnetic potential.) More probabilistic arguments can also be used to investigate such kind of problems, such as in the work of Goudon and Poupaud [14] for the transport case, or the work [15] of Komorowski, Novikov and Ryzhik, and the references therein, for the stochastic turbulence case. In the case of the heat equation driven by oscillatory potential one could think of a nice by-product of the celebrated work of Hairer on the KPZ equation [16] and a deep extension of it by Hairer, Pardoux and Piatniski [17]. These results are related with more probabilistic works concerning the same type of equations, as those of Gu and Bal [18] and Pardoux and Piatniski [19], for instance. In its ultimate form, rough paths ideas, such as used in the study of the KPZ equation, have grown into the theory of regularity structures [20]. On a more down-to-earth side, Kelly and Melbourne [21,22] have shown recently how one can use rough paths methods to investigate a fast–slow system of the form

$$\dot{x}_\epsilon = a(x_\epsilon, y_\epsilon) + \frac{1}{\epsilon} b(x_\epsilon, y_\epsilon),$$

where the dynamics of the fast component y_ϵ is autonomous and Anosov or axiom A, or even non-uniformly hyperbolic. We would like to put this result and some other homogenization results in the newly introduced setting of *rough flows* [23], that encompasses a large part of the theory of rough differential equations, and unifies it with the theory of stochastic flows. We provide for that purpose an easily usable black box for the study of homogenisation of random ordinary differential equations, under the form of a result

Convergence of finite dimensional marginals $\oplus$ *Moment/tightness bounds*

(for the driving vector fields)

$\Longrightarrow$ *Homogenisation* (for the dynamics)

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