ARTICLE IN PRESS YJDEQ:8862

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ●●● (●●●●) ●●●–●●●

Journal of Differential Equations

www.elsevier.com/locate/jde

Wong–Zakai approximations and center manifolds of stochastic differential equations ☆

Jun Shen [a], Kening Lu [a,b,*]

[a] *School of Mathematics, Sichuan University, Chengdu, Sichuan 610064, PR China*
[b] *Department of Mathematics, Brigham Young University, Provo, UT 84602, USA*

Received 14 March 2017

Abstract

In this paper, we study the Wong–Zakai approximations given by a stationary process via the Wiener shift and their associated dynamics of the stochastic differential equation driven by a l-dimensional Brownian motion. We prove that the solutions of Wong–Zakai approximations converge in the mean square to the solutions of the Stratonovich stochastic differential equation. We also show that for a simple multiplicative noise, the center-manifold of the Wong–Zakai approximations converges to the center-manifold of the Stratonovich stochastic differential equation.

MSC: primary 60H10; secondary 37D10, 37H10

Keywords: Brownian motion; Wong–Zakai approximations; Multiplicative noise; Random dynamical systems; Center manifolds

1. Introduction

In this paper, we study the Wong–Zakai approximations given by a stationary process via the Wiener shift and their associated dynamics of the following stochastic differential equation in $\mathbb{R}^n$:

☆ This work was supported by NSFC (11501549, 11331007) and NSF (1413603).
* Corresponding author.
E-mail addresses: junshen85@163.com (J. Shen), klu@math.byu.edu (K. Lu).

http://dx.doi.org/10.1016/j.jde.2017.06.005
0022-0396/© 2017 Published by Elsevier Inc.

Please cite this article in press as: J. Shen, K. Lu, Wong–Zakai approximations and center manifolds of stochastic differential equations, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.06.005

$$du = f(u)\,dt + \sigma(u) \circ dW, \tag{1.1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ and $\sigma : \mathbb{R}^n \to \mathbb{R}^{n\times l}$ are nonlinear functions, $W(t,\omega)$ is a l-dimensional Brownian motion, and $\circ\, dW(t,\omega)$ denotes the Stratonovich differential.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be the classical Wiener probability space, where

$$\Omega = C_0(\mathbb{R}, \mathbb{R}^l) := \{\omega \in C(\mathbb{R}, \mathbb{R}^l) : \ \omega(0) = 0\}$$

with the open compact topology, $\mathcal{F}$ is its Borel σ-algebra, and $\mathbb{P}$ is the Wiener measure. The Brownian motion has the form $W(t,\omega) = \omega(t)$. Consider the Wiener shift θ_t defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ by

$$\theta_t\omega(\cdot) = \omega(t + \cdot) - \omega(t).$$

It is known that the probability measure $\mathbb{P}$ is an ergodic invariant measure for θ_t. $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ forms a metric dynamical system, see Arnold [2].

For each $\delta > 0$, let $\mathcal{G}_\delta : \Omega \to \mathbb{R}^l$ denote the random variable

$$\mathcal{G}_\delta(\omega) = \frac{1}{\delta}\omega(\delta).$$

Then we have

$$\mathcal{G}_\delta(\theta_t\omega) = \frac{1}{\delta}(\omega(t+\delta) - \omega(t)). \tag{1.2}$$

From the properties of Brownian motions, it follows that $\mathcal{G}_\delta(\theta_t\omega)$ is a stationary stochastic process with a normal distribution and is unbounded in t for almost all ω. $\mathcal{G}_\delta(\theta_t\omega)$ may be viewed as an approximation of white noise in the sense

$$\lim_{\delta\to 0^+} \sup_{t\in[0,T]} \left| \int_0^t \mathcal{G}_\delta(\theta_s\omega)ds - \omega(t) \right| = 0, a.s.$$

for each $T > 0$, which will be proved in Section 3.

We consider the following Wong–Zakai approximation of equation (1.1) driven by a multiplicative noise of $\mathcal{G}_\delta(\theta_t\omega)$:

$$\dot{u}_\delta = f(u_\delta) + \sigma(u_\delta)\mathcal{G}_\delta(\theta_t\omega). \tag{1.3}$$

Note that the above equation is a random differential equation driven by the stationary stochastic process $\mathcal{G}_\delta(\theta_t\omega)$. As a consequence, its solutions generate a random dynamical system. Thus one can study its sample-wise (or pathwise) dynamical properties.

In current paper, we first study the limit behavior of solutions of equation (1.3) as $\delta \to 0^+$ and show that u_δ converges in the mean square to a solution of equation (1.1). Then, to illustrate that the dynamics of this Wong–Zakai approximations converge to ones of the stochastic equation (1.3), we show that for a simple multiplicative noise, the center-manifold of (1.3) converges to the center-manifold of stochastic equation (1.1).