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Impulse output rapid stabilization for heat equations

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Abstract

The main aim of this paper is to provide a new feedback law for the heat equations in a bounded domain Ω with Dirichlet boundary condition. Two constraints will be compulsory: First, the controls are active in a subdomain of Ω and at discrete time points; Second, the observations are made in another subdomain and at different discrete time points. Our strategy consists in linking an observation estimate at one time, minimal norm impulse control, approximate inverse source problem and rapid output stabilization.

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Keywords: Rapid output stabilization; Impulse control; Heat equation

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1. Introduction

Let $\Omega \subset \mathbb{R}^d$, $d \geq 1$, be a bounded domain, with a C^2 boundary $\partial\Omega$. Let V be a function in $L^\infty(\Omega)$ with its norm $\|\cdot\|_\infty$. Define

$$A := \Delta - V, \text{ with } D(A) = H^2(\Omega) \cap H_0^1(\Omega) \ .$$

Write $\{e^{tA}, t \geq 0\}$ for the semigroup generated by A on $L^2(\Omega)$. It is well-known that when $V = 0$, the semigroup $\{e^{tA}, t \geq 0\}$ has the exponential decay with the rate α_1, which is the first eigenvalue of $-\Delta$ with the homogeneous Dirichlet boundary condition. The aim of this study is to build up, for each $\gamma > 0$, an output feedback law $\mathcal{F}_\gamma$ so that any solution to the closed-loop controlled heat equation, associated with A and $\mathcal{F}_\gamma$, has an exponential decay with the rate γ. Further, two constraints are imposed: We only have accesses to the system at time $\frac{1}{2}T + \overline{\mathbb{N}}T$ on an open subdomain $\omega_1 \subset \Omega$; We only can control at time $T + \overline{\mathbb{N}}T$ on another open subset $\omega_2 \subset \Omega$. Here, T is an arbitrarily fixed positive number and $\overline{\mathbb{N}} := \mathbb{N} \cup \{0\}$, with $\mathbb{N}$ the set of all positive natural numbers. Therefore, the closed-loop controlled equation under consideration reads:

$$\begin{cases} y'(t) - Ay(t) = 0 \,, & t \in (0,\infty) \setminus \mathbb{N}T \,, \\ y(0) \in L^2(\Omega) \,, & \\ y(nT) = y(nT_-) + 1_{\omega_2}\mathcal{F}_\gamma\left(1_{\omega_1}^* y\left(\left(n - \frac{1}{2}\right)T\right)\right) \,, & \forall n \in \mathbb{N} \,. \end{cases} \tag{1.1}$$

Here, $y(nT_-)$ denotes the left limit of the function: $t \mapsto y(t)$ (from $[0,\infty)$ to $L^2(\Omega)$) at time nT; 1_{ω_2} denotes the zero-extension operator from $L^2(\omega_2)$ to $L^2(\Omega)$ (i.e., for each $f \in L^2(\omega_2)$, $1_{\omega_2}(f)$ is defined to be the zero-extension of f over Ω); $1_{\omega_1}^*$ stands for the adjoint operator of 1_{ω_1}; $\mathcal{F}_\gamma$ is a linear and bounded operator from $L^2(\omega_1)$ to $L^2(\omega_2)$. The operator $\mathcal{F}_\gamma$ is what we will build up. The evolution distributed system (1.1) is well-posed and can be understood as the coupling of a sequence of heat equations:

$$y(t) := \left| \begin{array}{ll} y^0(t) \,, & \text{if } t \in [0,T) \\ y^n(t) \,, & \text{if } t \in [nT,(n+1)T) \end{array} \right.$$

for any $n \in \mathbb{N}$, where

$$\begin{cases} \partial_t y^0 - \Delta y^0 + Vy^0 = 0 \,, & \text{in } \Omega \times (0,T) \,, \\ y^0 = 0 \,, & \text{on } \partial\Omega \times (0,T) \,, \\ y^0(0) = y(0) \in L^2(\Omega) \,, & \end{cases}$$

and

$$\begin{cases} \partial_t y^n - \Delta y^n + Vy^n = 0 \,, & \text{in } \Omega \times (nT,(n+1)T) \,, \\ y^n = 0 \,, & \text{on } \partial\Omega \times (nT,(n+1)T) \,, \\ y^n(nT) = y^{n-1}(nT) + 1_{\omega_2}\mathcal{F}_\gamma\left(1_{\omega_1}^* y^{n-1}\left(\left(n - \frac{1}{2}\right)T\right)\right) \in L^2(\Omega) \ . & \end{cases}$$

Throughout this paper, we denote by $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$ the norm and the inner product of $L^2(\Omega)$ respectively. The notation $\|\cdot\|_{\omega_1}$ and $\langle\cdot,\cdot\rangle_{\omega_1}$ will mean the norm and the inner product of $L^2(\omega_1)$

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