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On analyticity of linear waves scattered by a layered medium

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Abstract

The scattering of linear waves by periodic structures is a crucial phenomena in many branches of applied physics and engineering. In this paper we establish rigorous analytic results necessary for the proper numerical analysis of a class of High-Order Perturbation of Surfaces methods for simulating such waves. More specifically, we prove a theorem on existence and uniqueness of solutions to a system of partial differential equations which model the interaction of linear waves with a multiply layered periodic structure in three dimensions. This result provides hypotheses under which a rigorous numerical analysis could be conducted for recent generalizations to the methods of Operator Expansions, Field Expansions, and Transformed Field Expansions.

MSC: 35J25; 35Q60; 35Q86; 65N35; 78A45; 78M22

Keywords: High-Order Perturbation of Surfaces methods; Layered media; Linear wave scattering; Helmholtz equation; Diffraction gratings

1. Introduction

The scattering of linear waves by periodic structures (both in two and three dimensions) is a crucial phenomena in many branches of applied physics and engineering. From acoustics (e.g., remote sensing [69], nondestructive testing [67], and underwater acoustics [14]), to electromag-

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netics (e.g., extraordinary optical transmission [31], surface enhanced spectroscopy [52], and surface plasmon resonance biosensing [38,44]), to elastodynamics (e.g., full waveform inversion [70,18] and hazard assessment [36,68]), examples abound. Obviously, the ability to rapidly simulate such configurations numerically with great accuracy and high fidelity is of the upmost importance to many disciplines.

The most popular approaches to these problems in the engineering literature are *volumetric* numerical methods. For instance, sampling from the seismic imaging community alone, formulations based upon Finite Differences [64], Finite Elements [73], and Spectral Elements [41] are common. However, these methods suffer from the requirement that they discretize the full volume of the problem domain which results in both a prohibitive number of degrees of freedom for the layered structures we consider here, and the difficult question of appropriately specifying far-field boundary conditions explicitly.

For these reasons, *surface* methods are an appealing alternative, particularly Boundary Integral Methods [23,43,66,3,12,13,10,45,19] and the High-Order Perturbation of Surfaces (HOPS) methods of Milder (Operator Expansions) [47,48] (see also [22]), Bruno and Reitich (Field Expansions) [15–17], and the author and Reitich (Transformed Field Expansions) [56–58]. These surface methods are greatly advantaged over the volumetric algorithms discussed above primarily in the greatly reduced number of degrees of freedom required to resolve a computation, in addition to the *exact* enforcement of far-field boundary conditions. Consequently, these approaches are an extremely important alternative and are becoming more widely used by practitioners.

Of course there has been a huge amount of rigorous analysis on the systems of partial differential equations which model these scattering phenomena, in addition to the design of computational schemes. Most of these results utilize either Integral Equations techniques or weak formulations of the volumetric problem (each of which naturally lead to numerical implementations). We find the Habilitationsschrift of T. Arens [5] a particularly readable and definitive reference for the periodic layered media problems we consider here. In particular, we point the interested reader to Chapter 1 which discusses in great detail the state-of-the-art in both two and three dimensions for solutions of the Helmholtz and Maxwell equations. To summarize, in two dimensions most of the questions of existence and uniqueness have been satisfactorily addressed and these results are summarized in surveys such as those of Petit [62] and Bao, Cowsar, and Masters [7]. For single layer configurations we point out the work of Alber [2], Wilcox [71], and Elschner and Schmidt [32]. In three dimensions, for the Helmholtz equation, most results are connected to variational formulations such as those of Abboud and Nedelec [4], Bao [6], Bao, Dobson, and Cox [9], and Dobson [30] (see also the work of Chen and Friedman [21] and Dobson and Friedman [28] in the context of Maxwell's equations). Arens summarizes these with the following sentence [5]: "There may exist at most a countable set of frequencies with infinity as the only possible accumulation point for which the problem is not uniquely solvable."

The purpose of this contribution is to establish rigorous analytic results necessary for the proper numerical analysis of HOPS methods. More specifically, we prove a result (Theorem 4.1) using boundary perturbations on existence and uniqueness of solutions to a system of Helmholtz equations which model the interaction of linear (acoustic) waves with a multiply layered periodic structure in three dimensions. The goal of this study is to provide hypotheses under which a rigorous numerical analysis could be conducted, and a solution to which our HOPS schemes can be shown to converge. More specifically, we seek a framework to study the generalizations to the Operator Expansions method of the author and Fang [54,35], to the Field Expansions approach by the author and Malcolm [50,49,51,55], and, based upon the recursions derived herein, to the Transformed Field Expansions approach. For the numerical analysis, we have in mind an

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