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The Cauchy problem for the radially symmetric homogeneous Boltzmann equation with Shubin class initial datum and Gelfand–Shilov smoothing effect

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Abstract

In this paper, we study the Cauchy problem for the radially symmetric homogeneous non-cutoff Boltzmann equation with Maxwellian molecules, the initial datum belongs to Shubin space of the negative index which can be characterized by spectral decomposition of the harmonic oscillator, and it is a small perturbation of Maxwellian distribution. The Shubin space of the negative index contains the probability measures. Based on this spectral decomposition, we construct the weak solution with Shubin class initial datum, we also prove that the Cauchy problem enjoys Gelfand–Shilov smoothing effect, meaning that the smoothing properties are the same as the Cauchy problem defined by the evolution equation associated to a fractional harmonic oscillator.

MSC: 35Q20; 35E15; 35B65

Keywords: Cauchy problem; Boltzmann equation; Gelfand–Shilov smoothing effect; Shubin class initial datum

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Contents

1. Introduction

In this work, we consider the spatially homogeneous Boltzmann equation

$$\begin{cases} \partial_t f = Q(f, f), \\ f|_{t=0} = f_0 \geq 0, \end{cases} \tag{1.1}$$

where $f = f(t, v)$ is the density distribution function depending on the variables $v \in \mathbb{R}^3$ and the time $t \geq 0$. The Boltzmann bilinear collision operator is given by

$$Q(g, f)(v) = \int_{\mathbb{R}^3} \int_{\mathbb{S}^2} B(v - v_*, \sigma)(g(v_*')f(v') - g(v_*)f(v))dv_* d\sigma,$$

where for $\sigma \in \mathbb{S}^2$, the symbols v_*' and v' are abbreviations for the expressions,

$$v' = \frac{v + v_*}{2} + \frac{|v - v_*|}{2}\sigma, \quad v_*' = \frac{v + v_*}{2} - \frac{|v - v_*|}{2}\sigma,$$

which are obtained in such a way that collision preserves momentum and kinetic energy, namely

$$v_*' + v' = v + v_*, \quad |v_*'|^2 + |v'|^2 = |v|^2 + |v_*|^2.$$

The non-negative cross section $B(z, \sigma)$ depends only on $|z|$ and the scalar product $\frac{z}{|z|} \cdot \sigma$. For physical models, it usually takes the form

$$B(v - v_*, \sigma) = \Phi(|v - v_*|)b(\cos\theta), \quad \cos\theta = \frac{v - v_*}{|v - v_*|} \cdot \sigma, \ \ 0 \leq \theta \leq \frac{\pi}{2}.$$

Throughout this paper, we consider the Maxwellian molecules case which corresponds to the case $\Phi \equiv 1$ and focus our attention on the following general assumption of b

$$\beta(\theta) = 2\pi b(\cos 2\theta) \sin 2\theta \approx \theta^{-1-2s}, \text{ when } \theta \to 0^+, \tag{1.2}$$

for some $0 < s < 1$. Without loss of generality, we may assume that $b(\cos\theta)$ is supported on the set $\cos\theta \geq 0$. See for instance [7] for more details on $\beta(\cdot)$ and [22] for a general collision kernel.

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