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On the 3-D Vlasov–Poisson system with point charges: Global solutions with unbounded supports and propagation of velocity-spatial moments

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Abstract

A new energy functional, which is shown to decay to zero as time tends to infinite, is introduced for the three-dimensional Vlasov–Poisson plasma in the presence of heavy point charges with repulsive interaction. Moreover, a new kind of pointwise energy of a plasma particle relative to each point charge is also constructed and the variation of its root is shown to be controlled by the electrostatic fields. Based on those results, we prove global existence and uniqueness of a classical solution possibly having infinite kinetic energy, but with compact "velocity-spatial support" specified in Theorem 1.1. Furthermore, for the single point charge-Vlasov–Poisson plasma, global existence and polynomial propagation of "velocity-spatial moments" of weak solutions are also established.

MSC: 35F25; 35J05; 35Q83; 82C40

Keywords: Vlasov–Poisson system; Point charges; Velocity-spatial support; Global existence; Propagation of moments

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1. Introduction

In this paper, we investigate the dynamics of a plasma under the influence of N heavy point charges in three dimensional space. So charged particles in the plasma are subjected to two forces: one is the self-consistent electrostatic force $E = E(x,t)$, and the other is the Coulomb's force field $F = F(x,t)$ induced by the heavy point charges. In this context, let $f(x,v,t)$ be the phase space density of the plasma's particles at time $t \geq 0$, located at $x \in \mathbb{R}^3$ and moving with velocity $v \in \mathbb{R}^3$, then the plasma evolves according to the Vlasov–Poisson system:

$$\partial_t f + v \cdot \nabla_x f + (E + F) \cdot \nabla_v f = 0, \tag{1.1}$$

$$E(x,t) = \int_{\mathbb{R}^3} \frac{x-y}{|x-y|^3} \rho(t,y) dy, \quad \rho(x,t) = \int_{\mathbb{R}^3} f(x,v,t) dv, \tag{1.2}$$

$$F(x,t) = \sum_{\alpha=1}^{N} \frac{x - \xi_\alpha(t)}{|x - \xi_\alpha(t)|^3}, \tag{1.3}$$

where $\rho(x,t)$ is the spatial density of charges in the plasma, and $\xi_\alpha(t) \in \mathbb{R}^3$ is the location of the α-th heavy point charge at time $t \geq 0$. If $U(x,t)$ solves the Poisson equation

$$-\Delta U(x,t) = \rho(x,t), \quad \lim_{|x|\to\infty} U(x,t) = 0,$$

then $E(x,t) = -\nabla_x U(x,t)$, where $U(x,t)$ the Coulomb's potential.

The motion of the α-th heavy point charge is itself governed by the second Newton law:

$$\dot{\xi}_\alpha(t) = \eta_\alpha(t), \qquad \dot{\eta}_\alpha(t) = E(\xi_\alpha(t), t) + F_\alpha(\xi_\alpha(t), t), \qquad \alpha = 1, 2, \cdots, N, \tag{1.4}$$

in which $\eta_\alpha(t) \in \mathbb{R}^3$ is the velocity of the corresponding particle and

$$F_\alpha(\xi_\alpha(t), t) = \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^{N} \frac{\xi_\alpha(t) - \xi_\beta(t)}{|\xi_\alpha(t) - \xi_\beta(t)|^3}.$$

So, we are led to consider the coupling evolutional system (1.1)–(1.4), which is called "Vlasov–Poisson system with point charges" or "plasma-charge model" through out this paper. In the following, we assume the initial state of the plasma-charge system is given by

$$f(x,v,0) = f_0(x,v); \qquad (\xi_\alpha(0), \eta_\alpha(0)) = (\xi_{\alpha 0}, \eta_{\alpha 0}), \qquad \alpha = 1, 2, \cdots, N. \tag{1.5}$$

It should be pointed out that mass and charge of all heavy particles have been normalized to one as indicated in (1.3) and (1.4). On the other hand, we have assumed that the plasma as well as all heavy particles are positively charged so that interactions between them are repulsive. If the plasma and the heavy particles have opposite sign, namely if we replace F by $-F$ in (1.1) and $E(\xi_\alpha(t), t)$ by $-E(\xi_\alpha(t), t)$ in (1.4) respectively, then they interact through attractive force, which may cause extra difficulties.