ARTICLE IN PRESS YJDEQ:8907

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Spatial behavior of solutions to the time periodic Stokes system in a three dimensional layer

K. Pileckas [a], M. Specovius-Neugebauer [b,*]

[a] Faculty of Mathematics and Informatics, Vilnius University, Naugarduko Str., 24, Vilnius, LT-03225 Lithuania
[b] Fachbereich 17 Mathematik, Universität Kassel, Heinrich Plett Str. 40, D-34132 Kassel, Germany

Received 16 November 2016; revised 23 June 2017

Abstract

Starting from solutions to the Dirichlet boundary value problem for the time-periodic Stokes system in a 3D-layer the subject of this paper is the asymptotic behavior of the velocity field and pressure for large x. For suitably decaying data the full asymptotic decomposition of the solution is constructed for large x and justified by estimates for the remainder. The decay conditions for the data and the estimates for the remainder are formulated in weighted Sobolev spaces with power-type weights.

MSC: 35Q30; 76D03; 76D05

Keywords: Time periodic Stokes system; Layer domain; Spatial asymptotic behavior

1. Introduction

We consider a time periodic Stokes flow

$$\partial_t \mathbf{u} - \Delta \mathbf{u} + \nabla p = \mathbf{f} \quad \text{in } I \times \Omega, \tag{1.1}$$

$$\operatorname{div} \mathbf{u} = g \quad \text{in } I \times \Omega, \tag{1.2}$$

* Corresponding author.
E-mail addresses: konstantinas.pileckas@mif.vu.lt (K. Pileckas), specovi@mathematik.uni-kassel.de (M. Specovius-Neugebauer).
URL: http://www.mathematik.uni-kassel.de/~specovi/ (M. Specovius-Neugebauer).

http://dx.doi.org/10.1016/j.jde.2017.07.014
0022-0396/

Please cite this article in press as: K. Pileckas, M. Specovius-Neugebauer, Spatial behavior of solutions to the time periodic Stokes system in a three dimensional layer, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.07.014

$$\mathbf{u}|_{z=0} = 0, \ u|_{z=1} = \mathbf{a} \quad \text{for all } t \in I, \tag{1.3}$$

$$\mathbf{u}|_{t=0} = \mathbf{u}|_{t=2\pi} \quad \text{in } \Omega, \tag{1.4}$$

in the infinite layer

$$\Omega = \{x = (y, z) : y = (y_1, y_2) \in \mathbb{R}^2, 0 < z < 1\}. \tag{1.5}$$

The boundary $\partial\Omega$ of Ω consists of the two planes $z = 0$ and $z = 1$. We look for a time periodic velocity field $\mathbf{u} = \mathbf{u}(t, y, z)$, and a scalar pressure $p = p(t, y, z)$ solving (1.1)–(1.4) on the time interval $I = [0, 2\pi]$.

For simplicity, we assume that the given external forces $\mathbf{f}$, g and the boundary data $\mathbf{a}$ are as smooth as we need in the calculations, and, of course, are periodic in time.

Although there are numerous papers on the time decay of solutions to the Stokes and Navier–Stokes initial boundary value problems in various types of domains, only few results are devoted to the spatial decay. In [6] existence and maximal regularity in L^r-spaces were considered for the time periodic Stokes problem in the whole space $\mathbb{R}^n$. But to our knowledge there are no results concerning the spatial behavior at infinity of time periodic solutions in a layer. The behavior of solutions to the stationary Stokes and Navier–Stokes problem were investigated in [8–10,13, 11,16]. In particular, it was shown in [9] that for the stationary Stokes problem the asymptotic behavior of the solutions for large r can be expressed in terms of solutions $\mathbf{U}^{(k,\pm)}$, $P^{(k,\pm)}$ to the homogeneous problem in the punctured layer $\dot{\Omega} := \Omega \setminus \{x : |y| = 0\}$:

$$U_y^{(k,\pm)}(y, z) = \frac{1}{2} z(z-1)\nabla_y P^{(k,\pm)}(y), \quad U_z^{(k,\pm)}(y, z) = 0 \tag{1.6}$$

$$P^{(k,+)}(y) = \frac{1}{\sqrt{2\pi|k|}} r^k \cos k\phi, \quad P^{(k,-)}(y) = \frac{1}{\sqrt{2\pi|k|}} r^k \sin k\phi, \quad k = \pm1, \pm2, \ldots \tag{1.7}$$

$$P^{(0,+)}(y) = 1, \quad P^{(0,-)}(y) = -\frac{1}{\sqrt{2\pi}} \ln r, \tag{1.8}$$

where (r, ϕ) are polar coordinates in $\mathbb{R}^2$. In particular the pressure terms are independent of z and harmonic in $\mathbb{R}^2 \setminus \{0\}$.

By modifying the terms in (1.6)–(1.8) it is possible to construct a sequence of time periodic solutions to the homogeneous problem

$$\begin{aligned} \partial_t \mathcal{U} - \Delta \mathcal{U} + \nabla \mathcal{P} &= 0 \quad \text{in } I \times \dot{\Omega}, \\ \operatorname{div} \mathcal{U} &= 0 \quad \text{in } I \times \dot{\Omega}, \\ \mathcal{U}|_{z=0} = \mathcal{U}|_{z=1} &= 0 \quad \text{for all } t \in I, \\ \mathcal{U}|_{t=0} = \mathcal{U}|_{t=2\pi} & \quad \text{in } \dot{\Omega}. \end{aligned} \tag{1.9}$$

Thereby we use the following ansatz:

$$\mathcal{U}(t, x) = w(t, z)\nabla_y q(y), \quad \mathcal{P}(t, x) = -s(t) q(y) \tag{1.10}$$

Clearly for solutions of this type we still have $u_z = 0$ and the continuity equation implies

$$\operatorname{div} \mathcal{U} = \operatorname{div}_y \mathcal{U}_y = w(t, z)\Delta_y q(y) = 0.$$

Please cite this article in press as: K. Pileckas, M. Specovius-Neugebauer, Spatial behavior of solutions to the time periodic Stokes system in a three dimensional layer, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.07.014