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Wavefronts for a nonlinear nonlocal bistable reaction–diffusion equation in population dynamics ☆

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Abstract

The wavefronts of a nonlinear nonlocal bistable reaction–diffusion equation,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u^2(1 - J_\sigma * u) - du, \quad (t, x) \in (0, \infty) \times \mathbb{R},$$

with $J_\sigma(x) = (1/\sigma)J(x/\sigma)$ and $\int_{\mathbb{R}} J(x)dx = 1$ are investigated in this article. It is proven that there exists a $c_*(\sigma)$ such that for all $c \geq c_*(\sigma)$, a monotone wavefront (c, ω) can be connected by the two positive equilibrium points. On the other hand, there exists a $c^*(\sigma)$ such that the model admits a semi-wavefront $(c^*(\sigma), \omega)$ with $\omega(-\infty) = 0$. Furthermore, it is shown that for sufficiently small σ, the semi-wavefronts are in fact wavefronts connecting 0 to the largest equilibrium. In addition, the wavefronts converge to those of the local problem as $\sigma \to 0$.

MSC: 35K65; 35K40

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1. Introduction

In this work we study the nonlinear nonlocal reaction–diffusion equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u^2(1 - J_\sigma * u) - du \quad \text{in } (0, \infty) \times \mathbb{R}, \tag{1.1}$$

where $0 \le d < \frac{2}{9}$, $J_\sigma(x) = \frac{1}{\sigma} J(\frac{x}{\sigma})$ is a σ-parameterized nonnegative kernel with

$$J \in L^1(\mathbb{R}), \qquad \int_{\mathbb{R}} J(x)dx = 1$$

and

$$J_\sigma * u(x) = \int_{\mathbb{R}} J_\sigma(x-y)u(y)dy.$$

This equation has three constant solutions,

$$0, \quad a = \frac{1}{2}(1 - \sqrt{1-4d}), \quad A = \frac{1}{2}(1 + \sqrt{1-4d}).$$

The problem arises in population dynamics with nonlocal consumption of resources, for example in [7,19]. It is used to model the behavior of various biological phenomena such as emergence and evolution of biological species and the process of speciation. Actually, similar nonlocal structure in the reaction term appears also in describing the behavior of cancer cells with therapy as well as polychemotherapy and chemotherapy [16,17].

The reaction term $u^2(1 - J_\sigma * u) - du$ consists of the reproduction which is proportional to the square of the density, the available resources and the mortality. The nonlocal consumption of the resources $J_\sigma * u(x)$ describes the phenomenon that consumption at the space point x is determined by the individuals located in some area around this point, where J_σ represents the probability density function that describes the distribution of individuals.

For $J(x) = 1$, with a general nonlinearity, $u^\alpha(1 - \int u(x,t)dx)$ in the multi-dimensional case, the problem has been studied [9,10] in terms of the existence of the classical solutions both in bounded and unbounded domains correspondingly.

In the case of $J(x) = \delta(x)$, where $\delta(x)$ is the Dirac function, equation (1.1) becomes the so called Huxley equation, which is a classical reaction–diffusion equation. It has the same constant solutions, 0, a and A to the nonlocal problem. The existence of traveling waves has been studied extensively in the literature (see [15,4,5,8,12,20] among others). It's proved that there exists a minimum speed such that the traveling waves connecting a and A exist for all values of the speed greater than or equal to this minimum speed. While the traveling waves connecting 0 and A exist only for a single value of the speed.

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