ARTICLE IN PRESS YJDEQ:8911

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Journal of Differential Equations

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Controllability of impulse controlled systems of heat equations coupled by constant matrices

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Received 21 January 2017; revised 20 June 2017

Abstract

This paper studies the approximate and null controllability for impulse controlled systems of heat equations coupled by a pair (A, B) of constant matrices. We present a necessary and sufficient condition for the approximate controllability, which is exactly Kalman's controllability rank condition of (A, B). We prove that when such a system is approximately controllable, the approximate controllability over an interval $[0, T]$ can be realized by adding controls at arbitrary $q(A, B)$ different control instants $0 < \tau_1 < \tau_2 < \cdots < \tau_{q(A,B)} < T$, provided that $\tau_{q(A,B)} - \tau_1 < d_A$, where $d_A \triangleq \min\{\pi/|\mathrm{Im}\,\lambda| : \lambda \in \sigma(A)\}$ and $q(A, B) \le n$. We also show that in general, such systems are not null controllable.

MSC: 93B05; 35K40

Keywords: Impulse control; Approximate controllability; Null controllability; Systems of heat equations

1. Introduction

In this paper, we will study the null controllability and the approximate controllability for some impulse controlled systems of heat equations coupled by constant matrices. Impulse control

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1 The author was partially supported by the National Natural Science Foundation of China under grant 11571264.

http://dx.doi.org/10.1016/j.jde.2017.07.018

Please cite this article in press as: S. Qin, G. Wang, Controllability of impulse controlled systems of heat equations coupled by constant matrices, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.07.018

belongs to a class of important controls and has wide applications. In many cases impulse control is an interesting alternative to deal with systems that cannot be acted on by means of continuous control inputs, for instance, relevant control for acting on a population of bacteria should be impulsive, so that the density of the bactericide may change instantaneously, indeed continuous control would enhance drug resistance of bacteria (see [30] and [33]). Another application of impulse control in reality can be explained as follows: In materials science, quenching is the rapid cooling of a workpiece to obtain certain material properties. A type of heat treating, quenching prevents undesired low-temperature processes, such as phase transformations, from occurring. Formerly, a sequence of intermittent quenching is widely used in swordsmanship. We can regard such a quenching as an impulse control. Besides, there are many applications of impulse control theory to nanoelectronics (see Chapter 11 in [33]).

To introduce our controlled system, some notations are given in order. Let $\Omega \subset \mathbb{R}^N$ (with $N \in \mathbb{N}^+ \triangleq \{1, 2, \dots\}$) be a bounded domain with a C^2 boundary $\partial\Omega$. Let $\omega \subset \Omega$ be an open and nonempty subset with its characteristic function χ_ω. Write $\mathbb{R}^+ \triangleq (0, +\infty)$. Let A and B be respectively $n \times n$ and $n \times m$ (with $n, m \in \mathbb{N}^+$) real matrices, which are treated as linear operators from $\mathbb{R}^n$ and $\mathbb{R}^m$ to $\mathbb{R}^n$, respectively. Write $\mathbf{\Delta} \triangleq \operatorname{diag}\{\Delta, \dots, \Delta\}$ (where there are n Laplacians). Define

$$\mathcal{A} \triangleq \mathbf{\Delta} - A \ \text{ with } \ D(\mathcal{A}) \triangleq H^2(\Omega; \mathbb{R}^n) \cap H_0^1(\Omega; \mathbb{R}^n). \tag{1.1}$$

(Namely, for each $\mathbf{z} = (z_1, \dots, z_n)^\top \in D(\mathcal{A})$, with $z_i \in H^2(\Omega; \mathbb{R}) \cap H_0^1(\Omega; \mathbb{R})$, $i = 1, \dots, n$, we define $\mathcal{A}\mathbf{z} \triangleq \mathbf{\Delta}(z_1, \dots, z_n)^\top - A(z_1, \dots, z_n)^\top$.) One can easily check that $\mathcal{A}$ generates a C_0-semigroup $\{e^{\mathcal{A}t}\}_{t\geq 0}$ over $L^2(\Omega; \mathbb{R}^n)$. We treat χ_ω as a linear and bounded operator on $L^2(\Omega; \mathbb{R}^n)$ in the following manner: For each $\mathbf{z} = (z_1, \dots, z_n)^\top \in L^2(\Omega; \mathbb{R}^n)$ (where $z_k \in L^2(\Omega; \mathbb{R})$, $k = 1, \dots, n$), we define $\chi_\omega \mathbf{z} \triangleq (\chi_\omega z_1, \dots, \chi_\omega z_n)^\top$.

Consider the following impulse controlled system of heat equations:

$$\begin{cases} \partial_t \mathbf{y}(t) - \mathcal{A}\mathbf{y}(t) = 0, & t \in \mathbb{R}^+ \setminus \{\tau_k\}_{k=1}^p, \\ \mathbf{y}(\tau_k) - \mathbf{y}(\tau_k-) = \chi_\omega B \mathbf{u}_k, & k = 1, 2, \dots, p, \\ \mathbf{y}(0) = \mathbf{y}_0 \in L^2(\Omega; \mathbb{R}^n). \end{cases} \tag{1.2}$$

Here, $p \in \mathbb{N}^+$; $0 < \tau_1 < \cdots < \tau_p < \infty$, which are called control instants; $\mathbf{u}_k = (u_{k1}, \dots, u_{km})^\top$, $k = 1, \dots, p$, are taken from $L^2(\Omega; \mathbb{R}^m)$ and called impulse controls; $\mathbf{y}(\tau_k-)$ denotes the left limit at $t = \tau_k$ for the function $\mathbf{y}$. One can easily check that the equation (1.2) is well-posed. Write $\mathbf{y}(\cdot; \mathbf{y}_0, \{\tau_k\}_{k=1}^p, \{\mathbf{u}_k\}_{k=1}^p)$ for the unique solution of (1.2). It is clear that

$$\mathbf{y}(t; \mathbf{y}_0, \{\tau_k\}_{k=1}^p, \{\mathbf{u}_k\}_{k=1}^p) = e^{\mathcal{A}t}\mathbf{y}_0 + \sum_{1\leq k\leq p,\ \tau_k \leq t} e^{\mathcal{A}(t-\tau_k)} \chi_\omega B \mathbf{u}_k, \ t \geq 0. \tag{1.3}$$

Throughout this paper, $\|\cdot\|$ and $\langle \cdot, \cdot \rangle$ denote the usual norm and inner product of $L^2(\Omega; \mathbb{R}^n)$, respectively; A^*, B^* and $\mathcal{A}^*$ stand for the adjoint operators of A, B and $\mathcal{A}$, respectively; For each $C \in \mathbb{R}^{n\times n}$, we denote by $\sigma(C)$ the spectrum of C, and define

$$d_C \triangleq \min\Big\{\pi/|\operatorname{Im}\lambda| \ : \ \lambda \in \sigma(C)\Big\}. \tag{1.4}$$

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