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Hopf bifurcation in a reaction–diffusion equation with distributed delay and Dirichlet boundary condition ☆

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Abstract

The stability and Hopf bifurcation of the positive steady state to a general scalar reaction–diffusion equation with distributed delay and Dirichlet boundary condition are investigated in this paper. The time delay follows a Gamma distribution function. Through analyzing the corresponding eigenvalue problems, we rigorously show that Hopf bifurcations will occur when the shape parameter $n \geq 1$, and the steady state is always stable when $n = 0$. By computing normal form on the center manifold, the direction of Hopf bifurcation and the stability of the periodic orbits can also be determined under a general setting. Our results show that the number of critical values of delay for Hopf bifurcation is finite and increasing in n, which is significantly different from the discrete delay case, and the first Hopf bifurcation value is decreasing in n. Examples from population biology and numerical simulations are used to illustrate the theoretical results.
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1. Introduction

Reaction–diffusion models have been used to describe the spatiotemporal distribution of density functions of substances from particles, chemicals, organisms, to plants and animals in modeling biological and ecological systems [4,30,31]. In 1952, Alan Turing [40] proposed that spatial patterns in embryonic morphogenesis were driven by diffusion-induced instability. Since then, researchers in chemistry and developmental biology have successfully applied Turing theory to explain and simulate the patterns arisen in Hydra growth [12,28], pigmentation patterning in fish [25], spatial patterns in Chlorite-Iodide-Malonic Acid-starch chemical reaction [26], regulation of Hox gene in the transition of fins to limbs during evolution [32], to name just a few.

On the other hand, real biochemical or ecological dynamics often depends on the historical information of systems so time delays could occur in various modeling mechanisms, and the presence of time delay may have profound impact on the dynamics of reaction–diffusion models [6,17,42]. The delay effect to a scalar reaction–diffusion population model has been considered in, for example, [3,5,18,36,38,43]. In general, a larger delay destabilizes the stable steady state of the system and an oscillatory pattern arises from a Hopf bifurcation. The stable steady state under Neumann boundary condition is usually a constant one, thus the Hopf bifurcation analysis is relatively easier [29,44]. For Dirichlet boundary problem, a positive steady state is always spatially non-homogeneous which makes such analysis difficult. Following the approach in [3], Su et al. [36] considered a general scalar diffusive equation with delayed growth rate per capita and Dirichlet boundary condition:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d\dfrac{\partial^2 u(x,t)}{\partial x^2} + \lambda u(x,t) f(u(x,t-\tau)), & x \in (0,l),\ t>0, \\ u(0,t) = u(l,t) = 0, & t>0, \\ u(x,t) = \eta(x,t), & x \in (0,l),\ t \in [-\tau, 0], \end{cases}$$

where $d > 0$ is the diffusion coefficient, $\tau > 0$ represents the time delay, $\lambda > 0$ is a scaling constant. The nonlinear function f is the growth rate per capita which can be chosen properly so that this equation can embrace different kinds of population dynamics. They proved that the non-homogeneous positive steady state loses its stability when τ increases and analyzed associated Hopf bifurcations. In [43], Yan and Li extended the results to a higher dimensional domain and also proved the stability of the bifurcating periodic orbits.

The dependence of the rate of change of current population on the population at a particular point of past time is usually a simplified assumption, and a more reasonable dependence would be on the whole historical information of the population. A distributed delay has been proposed to describe the population growth of some species, which can date back to the work of Volterra [41]. Here we propose a diffusive population model with general growth rate incorporating a distributed delay and Dirichlet boundary condition based on the previous work in [36]:

$$\begin{cases} u_t(x,t) = d\Delta u(x,t) + \lambda u(x,t) f(g * u(x,t-s)), & x \in \Omega,\ t>0, \\ u(x,t) = 0, & x \in \partial\Omega,\ t>0, \\ u(x,t) = u_0(x,t), & x \in \Omega,\ t \in (-\infty, 0], \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbf{R}^k$ $(k \geq 1)$ with smooth boundary, and $u_0 \in \mathcal{C} \triangleq C((-\infty,0],Y)$ with $Y = L^2(\Omega)$. Here, $d > 0$ represents the diffusion coefficient, $\lambda > 0$ is a growth rate coeffi-

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