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Nonlinear thermal instability in the magnetohydrodynamics problem without heat conductivity

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Abstract

We investigate the nonlinear thermal instability of the magnetohydrodynamic problem for a full compressible viscous fluid with zero resistivity and zero heat conductivity in the presence of a uniform gravitational force in a bounded domain $\Omega \in \mathbb{R}^3$. We establish that under some instability conditions, the equilibrium-state is linearly unstable by constructing a suitable energy functional and employing the modified variational method. Then, on the basis of the constructed linearly unstable solutions and the local well-posedness of classical solutions to the original nonlinear problem, we reconstruct the initial data of linearly unstable solutions to be the one of the original nonlinear problem and obtain an appropriate energy estimate of Gronwall-type. Finally, we establish that the equilibrium-state is nonlinearly unstable in view of the energy estimate by a bootstrap method.

MSC: 76D03; 35Q35

Keywords: Thermal instability; Magnetohydrodynamic problem; Compressible viscous fluid

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1. Introduction

The three-dimensional (3D) full compressible and viscous magnetohydrodynamic (MHD) equations with zero resistivity and zero heat conductivity in the presence of a uniform gravitational field in a bounded domain $\Omega \in \mathbb{R}^3$ read as follows

$$\begin{cases} \rho_t + \mathrm{div}(\rho \mathrm{u}) = 0, \\ (\rho \mathrm{u})_t + \mathrm{div}(\rho \mathrm{u} \otimes \mathrm{u}) + \nabla P = \lambda_0 (\nabla \times M) \times M + \mathrm{div}\, \mathbb{S} - \rho g e_3, \\ \mathbb{E}_t + \mathrm{div}(u(\mathbb{E}' + P)) = \mathrm{div}(\lambda_0 (u \times M) \times M + u\mathbb{S}), \\ M_t = \nabla \times (u \times M), \\ \mathrm{div} M = 0, \end{cases} \tag{1}$$

where ρ, $u \in \mathbb{R}^3$, $P = P(\rho, \theta)$, $M \in \mathbb{R}^3$, $\mathbb{E}$ and θ denote the density, velocity, pressure, magnetic field, the total energy and the absolute temperature, respectively. Here, $\mathbb{E}$ is the total energy given by

$$\mathbb{E} = \mathbb{E}' + \frac{1}{2}|M|^2 \tag{2}$$

with $\mathbb{E}' = \rho\big(e + \frac{1}{2}|u|^2\big)$, and $\mathbb{S}$ is the stress tensor given by

$$\mathbb{S} = \mu(\nabla u + (\nabla u)^T) + \lambda(\mathrm{div} u)\mathbb{I}, \tag{3}$$

where $\mathbb{I}$ is the identity matrix, and $\mu > 0$ and λ are the coefficients of viscosity and the second coefficient of viscosity satisfying the usual condition $\lambda + \frac{2}{3}\mu \geq 0$, respectively. λ_0 stands for the permeability of vacuum, $g > 0$ for the gravitational constant, $e_3 = (0, 0, 1)^T$ for the vertical unit vector, and $-\rho g e_3$ for the gravitational force. In this paper, we will assume that the gas is ideal and satisfy the relationship $P = R\rho\theta$ and $e = c_V \theta$, where R is the gas constant, and c_V is the heat capacity of the gas at constant volume. Then the equations (1) read as

$$\begin{cases} \rho_t + \mathrm{div}(\rho \mathrm{u}) = 0, \\ (\rho \mathrm{u})_t + \mathrm{div}(\rho \mathrm{u} \otimes \mathrm{u}) + \nabla(P + \lambda_0 |M|^2/2) \\ = \mu \Delta u + (\mu + \lambda)\nabla \mathrm{div} u - \rho g e_3 + \lambda_0 M \cdot \nabla M, \\ (\rho\theta)_t + \mathrm{div}(\rho\theta u) + P \mathrm{div} u = \dfrac{1}{c_V}[\mu|\nabla u + (\nabla u)^T|^2 + \lambda(\mathrm{div} u)^2], \\ M_t = M \cdot \nabla u - u \cdot \nabla M - M \mathrm{div} u, \\ \mathrm{div} M = 0, \end{cases} \tag{4}$$

without loss of generality, we set $c_V = 1$.

Now, we consider the equilibrium state $(\rho, u, \theta, M) = (\bar{\rho}, 0, \bar{E}, \bar{M}_c)$. We choose a density profile $\bar{\rho} := \bar{\rho}(x_3)$, which is independent of (x_1, x_2) and satisfies

$$\bar{\rho} \in C^4(\bar{\Omega}), \quad \inf_{x \in \Omega} \bar{\rho} > 0, \tag{5}$$

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