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Multiple periodic solutions for Γ-symmetric Newtonian systems

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Abstract

The existence of periodic solutions in Γ-symmetric Newtonian systems $\ddot{x} = -\nabla f(x)$ can be effectively studied by means of the $\Gamma \times O(2)$-equivariant gradient degree with values in the Euler ring $U(\Gamma \times O(2))$. In this paper we show that in the case of Γ being a finite group, the Euler ring $U(\Gamma \times O(2))$ and the related basic degrees are effectively computable using Euler ring homomorphisms, the Burnside ring $A(\Gamma \times O(2))$, and the reduced $\Gamma \times O(2)$-degree with no free parameters. We present several examples of Newtonian systems with various symmetries, for which we show existence of multiple periodic solutions. We also provide exact value of the equivariant topological invariant for those problems.

MSC: 37J45; 37C25; 37C80; 37K05; 47H11; 55M25

Keywords: Newtonian systems; Periodic solutions; Euler ring; Equivariant degree; Variational problem

In this paper, we study the existence of non-constant p-periodic solutions in Γ-symmetric Newtonian systems (Γ is a finite group) of type

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$$\ddot{x}(t) = -\nabla f(x(t)), \quad x(t) \in V, \tag{1}$$

where $V := \mathbb{R}^n$ is an orthogonal Γ-representation and $f : V \to \mathbb{R}$ a C^2-differentiable Γ-invariant function. By time-rescaling, this problem can be reduced to the following one-parameter system:

$$\begin{cases} \ddot{x}(t) = -\lambda^2 \nabla f(x(t)), \ x(t) \in V, \\ x(0) = x(2\pi), \ \dot{x}(0) = \dot{x}(2\pi). \end{cases} \tag{2}$$

Many methods applied to (1) were inspired by the Hamiltonian systems of type

$$\dot{x}(t) = J\nabla f(x(t)), \quad x(t) \in V, \tag{3}$$

where $V = \mathbb{R}^{2n}$ and $J = \begin{pmatrix} 0 & -\mathrm{Id} \\ \mathrm{Id} & 0 \end{pmatrix}$ is the symplectic matrix, for which the existence of 2π-periodic solutions is intensively studied (see, for example, [1,3,7,15–19,29,30,35,33,34,36,37,39,40,43]). Similar methods are also developed for the system (1) (see [2,9,22,23,26,24,25,53]). After K. Gęba introduced the concept of the gradient equivariant degree (see [27]), which led to the development of new equivariant-theoretical methods, several interesting papers by A. Golebiewska, J. Fura, A. Ratajczak, W. Radzki, H. Ruan and S. Rybicki were published (see [21,20,31,42,45–48]). These authors applied the $(\Gamma \times SO(2))$-equivariant degree to study the existence of multiple 2π-periodic solutions to (1). However, there is a significant difference between systems (1) and (3): system (1) is time-reversible, so it leads to a variational problem with $(\Gamma \times O(2))$-symmetry. As the equivariant gradient degree provides *complete* equivariant topological classification of the critical set for the variational functional $\mathscr{J} : H^1(S^1; V) \to \mathbb{R}$ related to (1), it is important to consider the *full* symmetry group for $\mathscr{J}$. In addition, if the function f is even, then the functional $\mathscr{J}$ associated with (1) becomes $(\Gamma \times \mathbb{Z}_2 \times O(2))$-invariant. It is our strong conviction that in order to make the most efficient use of the equivariant degree method, one cannot ignore symmetric properties of $\mathscr{J}$ for the sake of computational simplifications.

It seems that the main difficulty in using the G-equivariant gradient degree method comes from the topological sophistication of the definition. The G-equivariant gradient degree takes values in the Euler ring $U(G)$, which was introduced by T. tom Dieck. In his monograph (see [51]), Dieck presented several cohomological formulae for the ring multiplication. However, for an arbitrary compact Lie group G, the computation may constitute an extraordinary challenge. Very often, users of the G-equivariant degree method are confined to only certain (simpler or well-worked out) types of symmetry group G due to the computational obstacles, which is, inevitably, resulting in ignoring the full symmetry for the related problem (see [42,28,4]).

Notice that $U(G)$ generalizes the Burnside ring $A(G)$, which is a part of $U(G)$. While the computation of $U(G)$ remains a difficult task, that in $A(G)$ is relatively easy.

One should point out that other algebraic structures were also used in the ranges of various versions of equivariant degrees. For instance, the twisted $(\Gamma \times S^1)$-equivariant degree with one parameter (which takes values in the $A(\Gamma)$-module $A_1(\Gamma \times S^1)$; see [6]) and the twisted $(\Gamma \times \mathbb{T}^n)$-equivariant degree with n-parameters (which takes values in $A(\Gamma)$-module $A_1(\Gamma \times \mathbb{T}^n)$; see [13]). These additional structures, for which there is a well established computational foundation, can be used to simplify the computation of the Euler ring $U(G)$.

Consider the Euler ring homomorphism $\Psi : U(\Gamma \times O(2)) \to U(\Gamma \times SO(2))$ induced by the inclusion $i : \Gamma \times SO(2) \hookrightarrow \Gamma \times O(2)$ (see [51,4]). In this paper, we will show that for $G := \Gamma \times O(2)$ (Γ is a finite group), the Euler ring $U(G)$ can be fully described by means of the

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