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The steady state collision of two compressible subsonic perfect flows ☆

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Abstract

This paper is devoted to the mathematical theory for the steady state collision of two compressible subsonic irrotational flows issuing from two infinitely long nozzles. We established the existence, uniqueness and asymptotic behavior of the subsonic collision flow with a smooth interface for the steady Euler system in an important physical regime. More precisely, there exists a critical value, if the summation of the incoming mass fluxes in the inlets of the nozzles is less than the critical value, then there exists a unique smooth subsonic compressible collision flow. And furthermore, there exists a smooth interface which separates the two non-miscible compressible subsonic flows. In particular, a key observation is that the location of the interface can be determined uniquely by the incoming mass fluxes in the inlets of the nozzles. Finally, we showed some monotonic relationship between the location of the interface and the incoming mass fluxes.

MSC: 35Q31; 35J25; 76G25; 76N10

Keywords: Subsonic flows; Steady Euler equations; Existence; Uniqueness; Free interface

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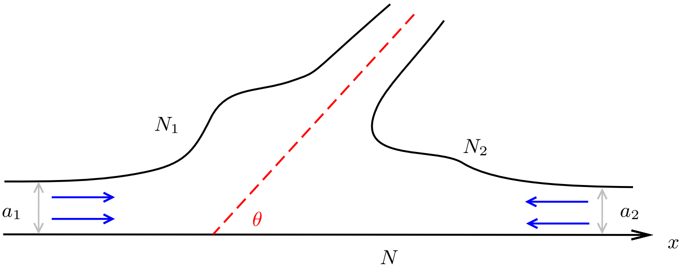

Fig. 1. Two infinitely long co-axis symmetric nozzles.

1. Introduction and main results

The collision dynamics of two compressible fluids issuing from two channels is of relevance to a variety of natural phenomena and technological applications, such as shaped charge (in [10]), spray combustion in gas-fueled combustors, mixing of slurry, etc. Any modeling of these flows requires a detailed understanding of the mechanics of the collision of the two-phase fluids. It's a quite challenging problem not only in fluid mechanics but also in mathematics, as suggested by some of its features, such as the complexity of the interface, the compressible effects, the possible presence of the singularity and so on.

This paper concerns the steady state collision of two non-miscible smooth subsonic potential flows issuing from two infinitely long symmetric nozzles (see Fig. 1). As usual, the two symmetric nozzles are assumed to be co-axis and symmetric respect to the x-axis. For convenience, we consider the upper part of the nozzles. The upper nozzle walls are denoted by N_1 and N_2, which can be described by two smooth functions $x = f_1(y)$ and $x = f_2(y)$, respectively.

Moreover, suppose that $f_i(y) \in C^{2,\alpha}((a_i, +\infty))$ for $a_i > 0$ $(i = 1, 2)$ and satisfies that

$$f_1(y) < f_2(y) \quad \text{for } y \geq \max\{a_1, a_2\}, \tag{1.1}$$

and

$$\lim_{y \to a_1^+} f_1(y) = -\infty, \qquad \lim_{y \to a_2^+} f_2(y) = +\infty. \tag{1.2}$$

Moreover, assume that $f_i(y)$ is a monotonic function as y approaches to a_i, i.e.

$$N_i \text{ can be described as } y = g_i(x), \text{ for } a_i < y < a_i + \varepsilon_i \text{ and some } \varepsilon_i > 0. \tag{1.3}$$

In the outlet, we assume that there exist some real constants k, b_1 and b_2 with $b_1 < b_2$, such that

$$f_i(y) - (ky + b_i) \to 0 \text{ and } f_i'(y) \to k, \text{ as } y \to \infty, \text{ for } i = 1, 2. \tag{1.4}$$

Hence, there exist an asymptotic direction $e = (\cos\theta, \sin\theta)$ with $\theta = \arctan \dfrac{1}{k}$ and an asymptotic distance between the two nozzle walls $d = (b_2 - b_1)\sin\theta$ as $y \to \infty$.

In this work, we seek a steady, non-miscible, subsonic two-phase fluid, which satisfies the following properties.

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