



A classical Perron method for existence of smooth solutions to boundary value and obstacle problems for degenerate-elliptic operators via holomorphic maps [☆]

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Abstract

We prove existence of solutions to boundary value problems and obstacle problems for degenerate-elliptic, linear, second-order partial differential operators with partial Dirichlet boundary conditions using a new version of the Perron method. The elliptic operators considered have a degeneracy along a portion of the domain boundary which is similar to the degeneracy of a model linear operator identified by Daskalopoulos and Hamilton [9] in their study of the porous medium equation or the degeneracy of the Heston operator [21] in mathematical finance. Existence of a solution to the partial Dirichlet problem on a half-ball, where the operator becomes degenerate on the flat boundary and a Dirichlet condition is only imposed on the spherical boundary, provides the key additional ingredient required for our Perron method. Surprisingly, proving existence of a solution to this partial Dirichlet problem with “mixed” boundary conditions on a half-ball is more challenging than one might expect. Due to the difficulty in developing a global Schauder estimate and due to compatibility conditions arising where the “degenerate” and “non-degenerate boundaries” touch, one cannot directly apply the continuity or approximate solution methods. However, in dimension two, there is a holomorphic map from the half-disk onto the infinite strip in the complex plane and one can extend this definition to higher dimensions to give a diffeomorphism from the half-ball onto the infinite “slab”. The solution to the partial Dirichlet problem on the half-ball can thus be converted to a partial Dirichlet problem on the slab, albeit for an operator which now has exponentially growing coefficients. The

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required Schauder regularity theory and existence of a solution to the partial Dirichlet problem on the slab can nevertheless be obtained using previous work of the author and C. Pop [16]. Our Perron method relies on weak and strong maximum principles for degenerate-elliptic operators, concepts of continuous subsolutions and supersolutions for boundary value and obstacle problems for degenerate-elliptic operators, and maximum and comparison principle estimates previously developed by the author [13].

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1. Introduction

Suppose $\mathcal{O} \subseteq \mathbb{H}$ is a domain (possibly unbounded) in the open upper half-space $\mathbb{H} := \mathbb{R}^{d-1} \times \mathbb{R}_+$, where $d \geq 2$ and $\mathbb{R}_+ := (0, \infty)$, and $\partial_1 \mathcal{O} := \partial \mathcal{O} \cap \mathbb{H}$ is the portion of the boundary $\partial \mathcal{O}$ of \mathcal{O} which lies in \mathbb{H} , and $\partial_0 \mathcal{O}$ is the interior of $\partial \mathbb{H} \cap \partial \mathcal{O}$, where $\partial \mathbb{H} = \mathbb{R}^{d-1} \times \{0\}$ is the boundary of

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