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Global boundedness in a quasilinear chemotaxis system with general density-signal governed sensitivity ☆

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Abstract

In this paper we study the global boundedness of solutions to the quasilinear parabolic chemotaxis system: $u_t = \nabla \cdot (D(u)\nabla u - S(u)\nabla\varphi(v))$, $0 = \Delta v - v + u$, subject to homogeneous Neumann boundary conditions and the initial data u_0 in a bounded and smooth domain $\Omega \subset \mathbb{R}^n$ ($n \geq 2$), where the diffusivity $D(u)$ is supposed to satisfy $D(u) \geq a_0(u+1)^{-\alpha}$ with $a_0 > 0$ and $\alpha \in \mathbb{R}$, while the density-signal governed sensitivity fulfills $0 \leq S(u) \leq b_0(u+1)^{\beta}$ and $0 < \varphi'(v) \leq \frac{\chi}{v^k}$ for $b_0, \chi > 0$ and $\beta, k \in \mathbb{R}$. It is shown that the solution is globally bounded if $\alpha + \beta < (1 - \frac{2}{n})k + \frac{2}{n}$ with $n \geq 3$ and $k < 1$, or $\alpha + \beta < 1$ for $k \geq 1$. This implies that the large k benefits the global boundedness of solutions due to the weaker chemotactic migration of the signal-dependent sensitivity at high signal concentrations. Moreover, when $\alpha + \beta$ arrives at the critical value, we establish the global boundedness of solutions for the coefficient χ properly small. It should be emphasized that the smallness of χ under $k > 1$ is positively related to the total cellular mass $\int_\Omega u_0 dx$, which is attributed to the stronger singularity of $\varphi(v)$ at $v = 0$ for $k > 1$ and the fact that v can be estimated from below by a multiple of $\int_\Omega u_0 dx$. In addition, distinctive phenomena concerning this model are observed by comparison with the known results.

MSC: 35B35; 35B40; 35K55; 92C17

Keywords: Chemotaxis; Quasilinear parabolic; Global boundedness; Signal-dependent sensitivity

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1. Introduction

In this paper, we consider the following quasilinear parabolic–elliptic Keller–Segel system

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u - S(u)\nabla\varphi(v)), & x \in \Omega,\ t>0, \\ 0 = \Delta v - v + u, & x \in \Omega,\ t>0, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega,\ t>0, \\ u(x,0) = u_0(x), & x \in \Omega \end{cases} \tag{1.1}$$

in a bounded and smooth domain $\Omega \subset \mathbb{R}^n$ $(n \ge 2)$, where $\partial/\partial\nu$ denotes the derivative with respect to the outer normal of $\partial\Omega$, and the initial data $u_0 \in C^{\mu}(\bar{\Omega})$ $(0 < \mu < 1)$ is nonnegative with $u_0 \not\equiv 0$.

The system (1.1) is a variant of the classical Keller–Segel model [23] that describes a biological process *chemotaxis* in which cells (with density u) migrate towards higher concentrations of a chemical signal v produced by cells themselves. It is exhibited that (1.1) involves nonlinear cell self-diffusion measured by $D(u)$, and more general chemotactic cross-diffusion mechanisms with the density-dependent sensitivity $S(u)$ and the signal-dependent sensitivity $\varphi(v)$. Since the pioneering work of Keller and Segel, considerable efforts have been devoted to identifying the interactions of the random self-diffusion and the chemotactic cross-diffusion on the blow-up or global boundedness of solutions. See the survey [4,19] and references therein.

In this paper we especially focus on the effect of the signal-dependent sensitivity on the global boundedness of solutions. As reference, let us briefly recall related literature.

(I) Parabolic–elliptic case

- Suppose that $\varphi(v) = \chi v$ with $\chi > 0$. For the case $S(u) = u$ and $D(u) = 1$, the solutions are bounded in time when $n = 1$, or $n = 2$ and $\int_\Omega u_0 dx < 4\pi/\chi$; whereas there may exist the blow-up solutions if $n = 2$ and $\int_\Omega u_0 dx > 4\pi/\chi$, or $n \ge 3$ with arbitrary value of $\int_\Omega u_0 dx$ (cf. [21,27,28,32] and references therein). Generally, it was proved that the solutions are globally bounded if $n = 2$, $S(u) = u$ and $D(u) \ge c(u+1)^{1+\varepsilon}$ [22], or for more general $D(u)$ and $S(u)$ satisfying [6]

$$\frac{S(u)}{D(u)} \le \begin{cases} cu^{-\varepsilon} & \text{when } n = 2, \\ cu^{-1-\varepsilon} & \text{when } n = 3, \end{cases}$$

  where c and ε are positive constants.
- Likewise, let $\varphi(v) = v$. When the second equation in (1.1) is replaced by $0 = \Delta v - \bar{m} + u$ with $\bar{m} := \frac{1}{|\Omega|}\int_\Omega u_0 dx$ denoting the mean mass of cells, under the hypothesis that $D(u) \simeq u^{-\alpha}$ and $S(u) \simeq u^{\beta}$ for large u with $\alpha \ge 0$ and $\beta \in \mathbb{R}$, Winkler and Djie [40] indicated that the solutions remain uniformly bounded in time if $\alpha + \beta < 2/n$, whereas the blow-up may occur when $\alpha + \beta > 2/n$ (see also [7] for the special case $S(u) = u$). Moreover, it was shown in [5] that the critical case $D(u) \simeq u^{\frac{n-2}{n}}$ with $S(u) = u$ belongs to the blow-up regime also.
- Next other signal-dependent sensitivity functions $\varphi(v)$ are concerned so as to reflect their roles in dominating the model. Now it is assumed that $S(u) = u$ and $D(u) = 1$. In the case of $\varphi(v) = \chi \log v$ $(\chi > 0)$, whether the solutions are global or not is up to the size of the coefficient χ. More precisely, it was shown in [30] that the radial solutions are globally

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