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On singular equations with critical and supercritical exponents

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Abstract

We study the problem

$$(I_\varepsilon)\begin{cases} -\Delta u - \dfrac{\mu u}{|x|^2} = u^p - \varepsilon u^q \quad \text{in} \quad \Omega, \\ u > 0 \quad \text{in} \quad \Omega, \\ u \in H_0^1(\Omega) \cap L^{q+1}(\Omega), \end{cases}$$

where $q > p \geq 2^* - 1$, $\varepsilon > 0$, $\Omega \subseteq \mathbb{R}^N$ is a bounded domain with smooth boundary, $0 \in \Omega$, $N \geq 3$ and $0 < \mu < \bar{\mu} := \left(\frac{N-2}{2}\right)^2$. We completely classify the singularity of solution at 0 in the supercritical case. Using the transformation $v = |x|^\nu u$, we reduce the problem (I_ε) to (J_ε)

$$(J_\varepsilon)\begin{cases} -div(|x|^{-2\nu}\nabla v) = |x|^{-(p+1)\nu} v^p - \varepsilon |x|^{-(q+1)\nu} v^q \quad \text{in} \quad \Omega, \\ v > 0 \quad \text{in} \quad \Omega, \\ v \in H_0^1(\Omega, |x|^{-2\nu}) \cap L^{q+1}(\Omega, |x|^{-(q+1)\nu}), \end{cases}$$

and then formulating a variational problem for (J_ε), we establish the existence of a variational solution v_ε and characterize the asymptotic behavior of v_ε as $\varepsilon \to 0$ by variational arguments when $p = 2^* - 1$.

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Contents

1. Introduction

In this paper, we consider the following family of singular problems:

$$\begin{cases} -\Delta u - \dfrac{\mu u}{|x|^2} = u^p - \varepsilon u^q & \text{in} \quad \Omega, \\ \qquad u > 0 & \text{in} \quad \Omega, \\ \qquad u \in H_0^1(\Omega) \cap L^{q+1}(\Omega), \end{cases} \tag{1.1}$$

and

$$\begin{cases} -div(|x|^{-2\nu}\nabla v) = |x|^{-(p+1)\nu} v^p - \varepsilon |x|^{-(q+1)\nu} v^q & \text{in} \quad \Omega, \\ \qquad v > 0 \quad \text{in} & \Omega, \\ \qquad v \in H_0^1(\Omega, |x|^{-2\nu}) \cap L^{q+1}(\Omega, |x|^{-(q+1)\nu}), \end{cases} \tag{1.2}$$