



# On singular equations with critical and supercritical exponents

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## Abstract

We study the problem

$$(I_\varepsilon) \begin{cases} -\Delta u - \frac{\mu u}{|x|^2} = u^p - \varepsilon u^q & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u \in H_0^1(\Omega) \cap L^{q+1}(\Omega), \end{cases}$$

where  $q > p \geq 2^* - 1$ ,  $\varepsilon > 0$ ,  $\Omega \subseteq \mathbb{R}^N$  is a bounded domain with smooth boundary,  $0 \in \Omega$ ,  $N \geq 3$  and  $0 < \mu < \bar{\mu} := \left(\frac{N-2}{2}\right)^2$ . We completely classify the singularity of solution at 0 in the supercritical case. Using the transformation  $v = |x|^\nu u$ , we reduce the problem  $(I_\varepsilon)$  to  $(J_\varepsilon)$

$$(J_\varepsilon) \begin{cases} -div(|x|^{-2\nu} \nabla v) = |x|^{-(p+1)\nu} v^p - \varepsilon |x|^{-(q+1)\nu} v^q & \text{in } \Omega, \\ v > 0 & \text{in } \Omega, \\ v \in H_0^1(\Omega, |x|^{-2\nu}) \cap L^{q+1}(\Omega, |x|^{-(q+1)\nu}), \end{cases}$$

and then formulating a variational problem for  $(J_\varepsilon)$ , we establish the existence of a variational solution  $v_\varepsilon$  and characterize the asymptotic behavior of  $v_\varepsilon$  as  $\varepsilon \rightarrow 0$  by variational arguments when  $p = 2^* - 1$ .

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## 1. Introduction

In this paper, we consider the following family of singular problems:

$$\begin{cases} -\Delta u - \frac{\mu u}{|x|^2} = u^p - \varepsilon u^q & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u \in H_0^1(\Omega) \cap L^{q+1}(\Omega), \end{cases} \quad (1.1)$$

and

$$\begin{cases} -\operatorname{div}(|x|^{-2\nu} \nabla v) = |x|^{-(p+1)\nu} v^p - \varepsilon |x|^{-(q+1)\nu} v^q & \text{in } \Omega, \\ v > 0 & \text{in } \Omega, \\ v \in H_0^1(\Omega, |x|^{-2\nu}) \cap L^{q+1}(\Omega, |x|^{-(q+1)\nu}), \end{cases} \quad (1.2)$$

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