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Global well-posedness for axisymmetric MHD system with only vertical viscosity

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Abstract

In this paper, we are concerned with the global well-posedness of a tri-dimensional MHD system with only vertical viscosity in velocity equation for the large axisymmetric initial data. By making good use of the axisymmetric structure of flow and the maximal smoothing effect of vertical diffusion, we show that $\sup_{2\le p<\infty}\int_0^t \frac{\|\partial_z u(\tau)\|_{L^p}^2}{p^{3/4}}\,\mathrm{d}\tau<\infty$. With this regularity for the vertical first derivative of velocity vector field, we further establish losing estimates for the anisotropy tri-dimensional MHD system to get the high regularity of (u,b), which guarantees that $\int_0^t\|\nabla u(\tau)\|_{L^\infty}\,\mathrm{d}\tau<\infty$. This together with the classical commutator estimate entails the global regularity of a smooth solution.

MSC: 35B33; 35Q35; 76D03; 76D05

Keywords: MHD system; Vertical viscosity; Global well-posedness; A losing estimate

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1. Introduction

The magneto-hydrodynamics (MHD) equations govern the dynamics of the velocity and the magnetic field in electrically conducting fluids such as plasmas and reflect the basic physics conservation laws. It has been at the center of numerous analytical, experimental and numerical investigations. The Cauchy problem of the tri-dimensional incompressible MHD system has the following form

$$\begin{cases} (\partial_t + u\cdot\nabla)u - \nu_x\partial_{xx}^2 u - \nu_y\partial_{yy}^2 u - \nu_z\partial_{zz}^2 u + \nabla p = (b\cdot\nabla)b, \quad (t,\mathbf{x})\in\mathbb{R}^+\times\mathbb{R}^3, \\ (\partial_t + u\cdot\nabla)b - \eta_x\partial_{xx}^2 b - \eta_y\partial_{yy}^2 b - \eta_z\partial_{zz}^2 b = (b\cdot\nabla)u, \\ \operatorname{div} u = \operatorname{div} b = 0, \\ (u,b)|_{t=0} = (u_0,b_0), \end{cases} \tag{1.1}$$

where $u = u(\mathbf{x},t)$ denotes the velocity of the fluid, $b = b(\mathbf{x},t)$ stands for the magnetic field and the scalar function $p = p(\mathbf{x},t)$ is pressure. The parameters $\nu_x, \nu_y, \nu_z, \eta_x, \eta_y, \eta_z$ are nonnegative constants. In addition, the initial data u_0 and b_0 satisfy $\operatorname{div} u_0 = \operatorname{div} b_0 = 0$ and $\mathbf{x} = (x, y, z)$.

In the bi-dimensional case, the constants ν_z and η_z in problem (1.1) become zero. For this case, if all parameters $\nu_x, \nu_y, \eta_x, \eta_y$ are positive, some results concerning on the global well-posedness for sufficiently smooth initial data (see for example [5,23]) were established in terms of the L^2-energy estimate. When the four parameters are zero, it reduces to an ideal MHD system. The global regularity of this system is still a challenging open problem. So, it has been a hot research topic to examine the intermediate cases where some of the four parameters are positive in the past few years. Recently, Cao and Wu in [4] showed that smooth solutions are global for system (1.1) with $\nu_x > 0$, $\nu_y = 0$, $\eta_x = 0$, $\eta_y > 0$ or $\nu_x = 0$, $\nu_y > 0$, $\eta_x > 0$, $\eta_y = 0$. More progress has also been made on several other partial dissipation cases of the bi-dimensional MHD equations. For system (1.1) with $\nu_x > 0$, $\eta_x > 0$ and $\nu_y = \eta_y = 0$, Cao, Dipendra and Wu [3] derived that the horizontal component of any solution admits a global (in time) bound in any Lebesgue space L^{2r} with $1 < r < \infty$ and the bound grows no faster than the order of $r \log r$ as r increases. In [4] and [13], system (1.1) with $\nu_x = \nu_y = 0$, $\eta_x = \eta_y > 0$ are shown to posse global H^1 weak solutions. However, the uniqueness of such weak solutions and a global H^2-bound remain unknown. Very recently, when $\eta_x = \eta_y = 0$, $\nu_x = \nu_y > 0$ in system (1.1), the global well-posedness by assuming that the initial data is close to a non-trivial steady state was investigated in [14,22] and [28]. The global well-posedness of the 2D MHD system with a velocity damping term is referred to [26].

Nevertheless, except that the initial data have some special structures, it is still not known whether or not the tri-dimensional Navier–Stokes system (when $b = 0$ in (1.1)) with large initial data has a unique global smooth solution (see [17] and references therein). For instance, by assuming that the initial data is axisymmetric without swirl, Ladyzhenskaya [15] and Ukhovskii and Yudovich [25] independently proved that weak solutions are regular for all time (see also [11]). Inspired by [11,15] and [25], more recent works are devoted to considering the axisymmetric Boussinesq or MHD system without swirl component of the velocity field. The global regularity results have been obtained for the axisymmetric Boussinesq system without swirl, when the dissipation only occurs in one equation or is present only in one direction (anisotropic dissipation) (see, e.g., [1,6,8,9,18,19,27]). While for the axisymmetric MHD system, the first result that a specific geometrical assumption allows global well-posedness was established by Lei in [12]. More precisely, under the assumption that u_θ, b_r and b_z are trivial, he showed that there exists a unique global solution if the initial data is smooth enough. Later on, Jiu and Liu [10]

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